Formula Synthesis of Maximal Mechanical Torque on Volume for Cylindrical Magnetic Coupler

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Abstract **– In this paper the influence of basic design parameters (proportionality coefficient *β* and permanent magnets height *h*) on the mechanical torque for cylindrical magnetic coupler with rounded permanent magnets is researched. Such cylindrical magnetic couplers are often used in pumps and liquid mixers. At the beginning the design parameters and their relevance are explained. The mechanical torque is calculated applying the software QuickField. In the data analysis and for the coupler comparison the division of maximal mechanical torque on volume is taken as the main characteristic. Because of the planned optimization for the calculated variants there were synthesized few formulas, from which the most suitable were found (for all researched design parameters' ranges). The formulas are synthesized using a program, which is created by Oskars Onzevs. The program for the formula synthesis is based on regression models, and the basic principles of it are explained. All the calculations for experimental and formula data are made for magnetic coupler's design with one pole pair number *p* that is equal to three.**

Keywords **– Data analysis, design optimization, finite element analysis, magnetic devices, permanent magnet machines.**

I. INTRODUCTION

A magnetic coupler (MC) is a mechanism that is used to transfer the mechanical torque without a contact of the two main parts called half couplings. The torque is transferred using attraction and repulsion forces from permanent magnets placed on both half couplings [1]. For a cylindrical MC [2-5] there is an inner half coupling and an outer half coupling, where the first one is placed into the second one (outer half coupling). The bases of the half couplings are made of steel. Permanent magnets (PMs) are placed on the bases. Magnets most often are made of rear earth alloys such as neodymium-iron-boron (Nd-Fe-B) or samarium-cobalt (Sm-Co) [6]. Main design parameters for a cylindrical MC cross-section are given in Fig. 1.

Three more parameters are used as design parameters: the axial length *l*, the pole pair number *p* and the proportionality coefficient *β*. The coefficient *β* is expressed in (1):

$$\beta = \frac{\alpha_{PM}}{(\alpha_{PM} + \alpha_\delta)},$$ (1)

where α_{PM} is the PM angle and α_δ is the angle of air gap between two PMs placed next to each other (Fig. 2). These both angles together shape a pole pitch.
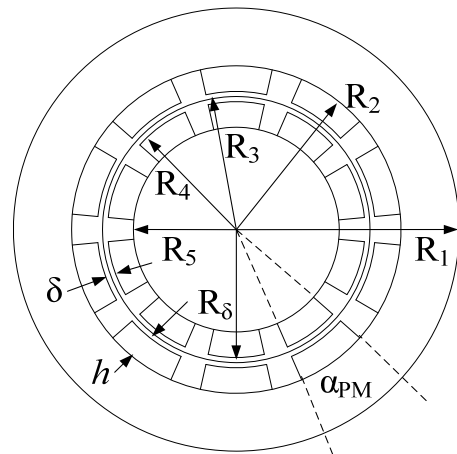

Fig. 1. Dimensions of cylindrical magnetic coupler in a cross-section. R_1-R_5 – radiuses; R_δ – radius of air gap's middle circle; δ – air gap; h – PMs height; α_{PM} – angle of PM.

In this paper multivariate experiments' data is obtained [7], which is analysed and formulas are synthesized, as well as the most suitable formula from the synthesized is chosen (for given data). The most suitable formula is tested by design parameters values which are between variants made for formula synthesis.

Such variables have been analysed: permanent magnets' height *h* (with values in millimetres – 6, 6.5, 7, 7.5, 8), proportionality coefficient *β* (with values – 0.7, 0.8, 0.9) and axial length *l* (with values in millimetres – 30, 32, 34, 36, 38).
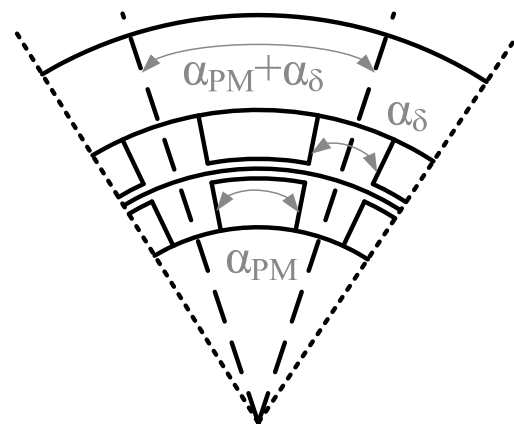

Fig. 2. The pole pitch, PM's angle and angle of the air gap. α_{PM} – angle of PM; α_δ – angle of air gap between two PMs placed next to each other.

The goal function is the maximal mechanical torque on the volume. Earlier research [8] was made just taking the maximal mechanical torque of a MC as the goal function. As the height of yoke is calculated (2), every change of PMs height *h*, coefficient *β* and pole pair number *p* lead to design changes and thus the influence is not only direct but also indirect on the maximal mechanical torque. Thus it is more correctly to choose the maximal mechanical torque on volume for the goal function. But, as the calculation of volume includes the axial length *l*, it is important to make a calculation set just with two independent variables: permanent magnets' height *h* and proportionality coefficient *β*.

$$h_{yoke} = \frac{\pi \cdot R_2 \cdot \beta}{p} \cdot \frac{B_{PM}}{B_{steel}} \qquad (2)$$

Other experiments showed that it is very complicated to obtain reliable synthesized formula for chosen interval of pole pair numbers: from 1 to 10, by a step equal one. Accordingly in this paper just one value of pole pair number interval ($p = 3$) was chosen to be used for finding out if in this case it would be easier to synthesize reliable formula for the goal function for planned optimization.

The formulas were also synthesized by the same variables for two control parameters: maximal magnetic flux density in steel of inner half coupling $B_{max\,in}$ and maximal magnetic flux density in the yoke of outer half coupling $B_{max\,out}$. Usually magnetic flux density maximal values are at the corners between the PM and the steel (Fig. 3).

And one more control parameter is the yoke of outer half coupling. It is necessary because in some cases it can be very small, e.g. 3.6 mm, but in the further planned optimization program for the best choice it will be controlled, that the yoke cannot be less than 5 mm – the reputed value for electrical machine yoke by physical strength [9].

The maximal mechanical torque of the MCs is calculated using the software QuickField [10]. Some parameters must be defined for the calculations. The PM's are made of rear earth
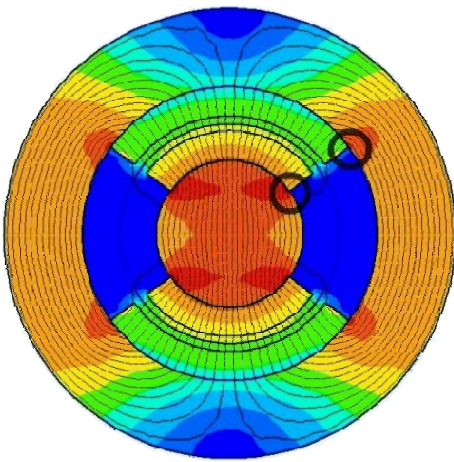

Fig. 3. The designed magnetic field for the magnetic coupler with p = 1. Two black circles mark the places, where the magnetic flux density has its highest value.

alloy neodymium-iron-boron and such parameters are used in the calculation:

- Residual magnetic flux density $B_r = 1.3$ (T);
- Coercive force $H_c = 900$ (kA/m);
- Relative permeability $\mu_* = 1.15$.

II. THE FORMULA SYNTHESIS

The program for formula synthesis is written by *Dr.sc.eng*. Oskars Onzevs. The formula synthesis is based on the regression models [11]. This work was further developed for engineering calculations [12] and so the methodology is successfully used for synthesis in the electrical machine field [13, 14]. The works of A. G. Ivakhnenko [15] determine the ideological criteria and the base of synthesis models.

The formula is synthesized as a sum of the elementary functions from the functions bank and mathematically it is expressed as (3):

$$Y = A_0 + \sum_{i=1}^{m-1} A_i g_i(X), \qquad (3)$$

where X – is the vector of independent variables, $g_i(X)$ – elementary functions selected from the functions bank, A_i – coefficients, which are obtained using the least squares method and m – the amount of elementary functions taken from the functions bank.

The elementary functions in the bank are given in form (4):

$$g_i(X) = \prod_{j=1}^{n} X_j^{a_{ij}}, \qquad (4)$$

where *n* is the number of independent variables and a_{ij} – index for every independent variable *j* in function *i* (can also be zero and negative).

For the functions synthesis there are two stages. In the first one a sufficient number of elementary functions are selected and (3) type relevance is obtained, which is more complicated than necessary. Every next elementary function is selected so, that the residual Δ would be described in the best way (5):

$$\Delta = A_0 + \sum_{i=1}^{m-1} A_i g_i(X) - Y. \qquad (5)$$

In the second stage the optimal relevance is found by complexity. It is done by cutting down the elementary functions one by one, which has the smallest influence on the inner criteria σ. Thus, in the elimination process, monotonous wane of inner criteria σ is obtained (6):

$$\sigma = \frac{1}{\sqrt{k-m+1}} \cdot \sqrt{\sum_{l=1}^{k} (A_0 + \sum_{i=1}^{m-1} A_i g_i(X) - Y)^2}, \quad (6)$$

where *k* – the number of experimental points.

To find the optimal complexity the outer criteria C (7) is calculated:

Electrical, Control and Communication Engineering

_____2013 / 3

$$C = \left(1 - \frac{\sigma}{\sigma_0}\right) \cdot 100\% , \qquad (7)$$

where

$$\sigma_0 = \sqrt{\frac{\sum_{l=1}^{k} \left(Y_l - \frac{1}{k}\sum_{i=1}^{k} Y_i\right)^2}{k-1}} .$$

The breaking point in relevance (8), here named as elimination diagram, determines the model of optimal complexity.

$$C = C(m) . \qquad (8)$$

III. SYNTHESIS FOR CHOSEN VARIABLES

First the formula synthesis was made for three variables: proportionality coefficient β, axial length l and PMs height h. When the relevance for $M_{max}/V = f(\beta, l, h)$ is researched, then the axial length is not an independent variable. That also shows up in the synthesized variants, where there are used only independent variables: coefficient β and PMs height h. Of the given five formulas one was chosen (9) to compare with the results given in *QuickField* for two variables.

$$M_{max}/V = 0.717 + 0.981\beta + 0.037h - 1.056\beta^2 - 0.001\beta \cdot h^2 \qquad (9)$$

The graphs for this formula and experimental data at different PMs height (6, 6.5, 7, 7.5 and 8 mm) at coefficients β = 0.7, β = 0.8 and β = 0.9 are given in fig. 5.
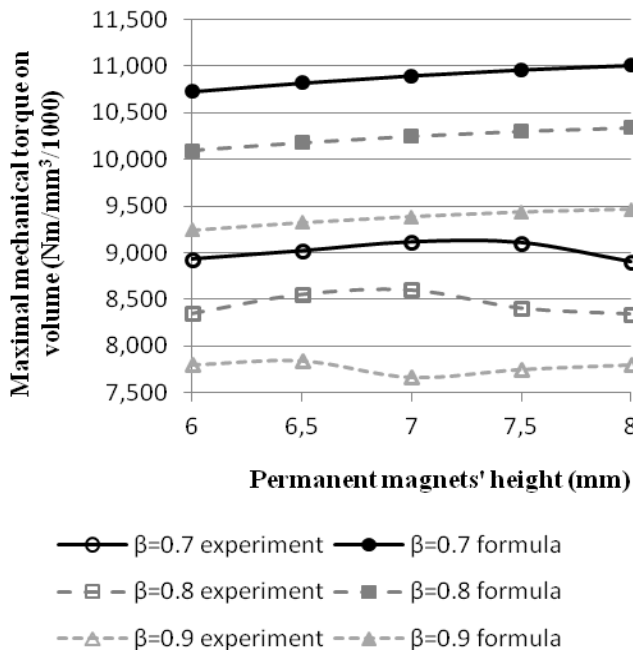
The difference for all coefficient cases is high enough, when expressed in percentage – the middle deviation is 20%. That is not acceptable. Further the results for synthesis by two independent variables are given.

From the six obtained formula variants one (10) was chosen to describe the main function – maximal mechanical torque on volume (Fig. 6). Experimental data is named "exper" and in some figures synthesized formula is named "formul" for short.
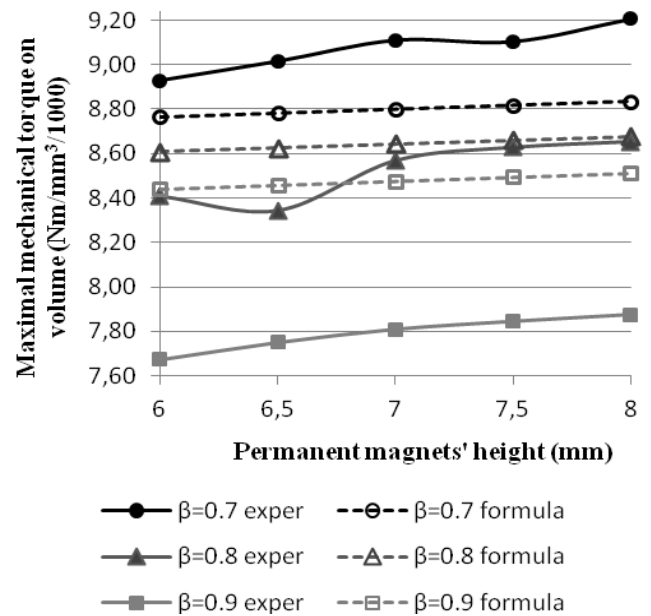

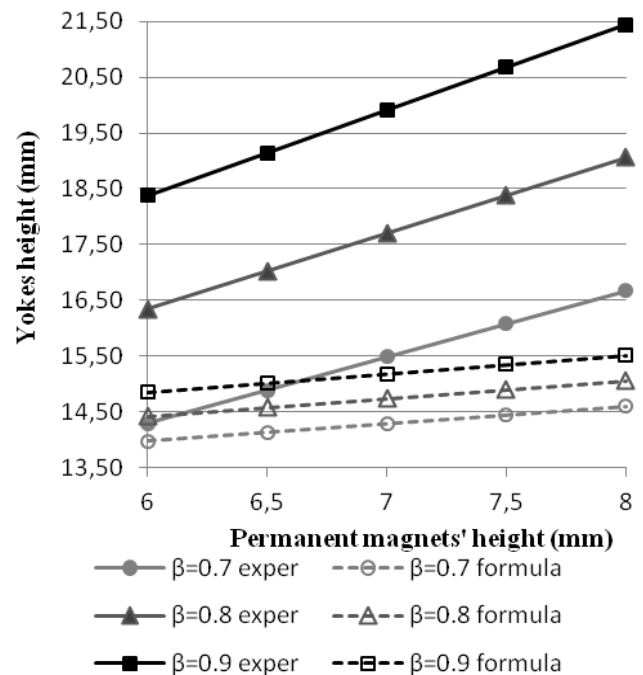Fig. 6. The function $M_{max}=f(h, \beta)$ (l = 30 mm).

Fig. 5. The function $M_{max}=f(h, \beta)$ for axial length l = 30 mm.

Fig. 7. The function $h_{yoke}=f(h, \beta)$ (l = 30 mm).

Electrical, Control and Communication Engineering

2013 / 3 _____

$$M_{\max}/V = 10.256 - 1.976\beta + 0.036h - 0.225\frac{1}{\beta} \quad (10)$$

For a smaller proportionality coefficient β ($\beta = 0.7$) there is a higher value of maximal mechanical torque on volume (Fig. 6). Also higher permanent magnets give better values of M_{max}/V. For this formula the deviation from experimental data is up to ten per cent, but the middle value of error is just four per cent, which is acceptable.

For the control parameter – yokes height h_{yoke} – nine formula variants were obtained, from which one was chosen. The results are compared with the experimental data (Fig. 7).

If compare the experimental and the results obtained by formula then the graphs are proportional to each other, but the best conformity is only in the case when the proportionality coefficient is $\beta = 0.7$.

For control parameters – maximal magnetic flux density in inner half coupling $B_{max\ in}$ and in outer half coupling $B_{max\ out}$ – four formula variants were synthesized for each. In this case also were compared the function values in *Excel* and one formula was chosen for both parameters. The chosen formula result for $B_{max\ in}$ is given for one case: proportionality coefficient $\beta = 0.7$ and axial length $l = 30$ mm (Fig. 8). The formula results give a linear function though in reality it is much more complicated. It is accepted till further research.

The chosen formula for $B_{max\ out}$ is given just for one proportionality coefficient ($\beta = 0.8$) and axial length ($l = 32$ mm) case (Fig. 9).

There are given just a few samples out of all obtained data. For all magnetic flux density values the program is finding the best possible relation (with the aim to obtain useful formula) to independent variable PMs height h (Fig. 8, Fig. 9) and to proportionality coefficient β. The maximal magnetic flux density (in inner and outer half couplings) value dependence on coefficient β is given in fig. 10.
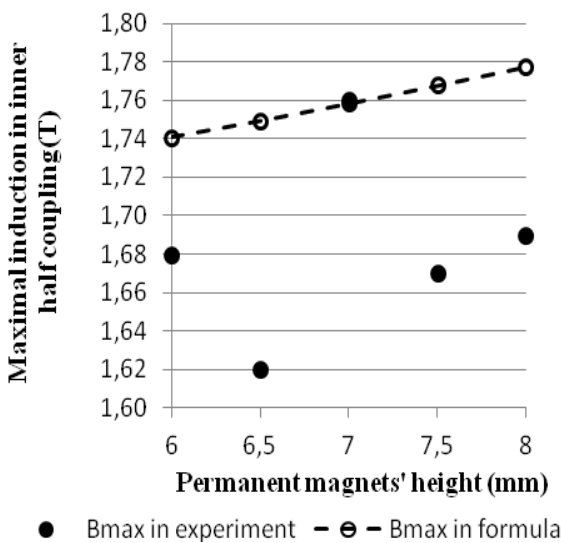
In this case the functions for maximal magnetic flux density are simpler than in reality, where the dependence is more complicated and is not linear, on the contrary as for the yokes height (Fig. 7).

IV. TESTING THE CHOSEN FORMULAS

Thirteen additional experiments were made to check, how the formulas behave for design variants, which are between the parameters values used for formula synthesis.

The additional variants are made in such principle:
1) The proportionality coefficients β are constant and PM height h is changed;
2) The PM heights h are constant as used for synthesis but proportionality coefficients β are varied;
3) Both variables (h, β) were varied with values, which were not used in formula synthesis.

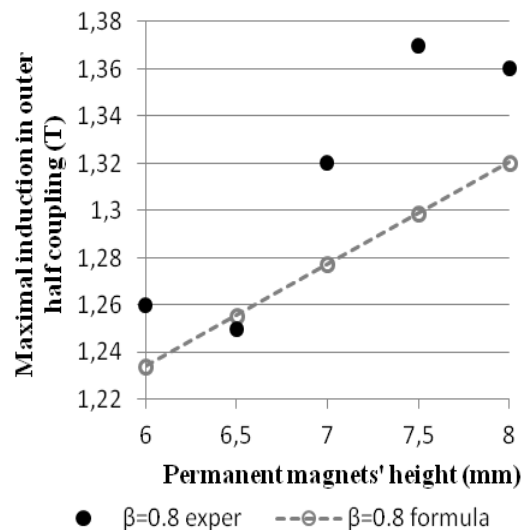Fig. 9. The function $B_{max\ out} = f(h)$, ($\beta = 0.8$, $l = 32$ mm).

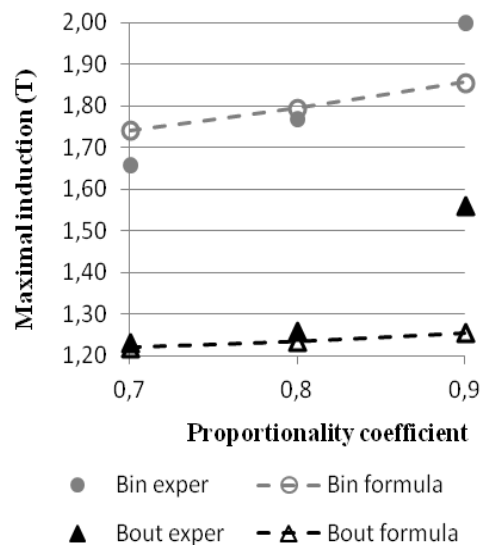Fig. 8. The function $B_{max\ in} = f(h)$, ($\beta = 0.7$, $l = 30$ mm).

Fig. 10. The function $B_{max\ in} = f(\beta)$, $B_{max\ out} = f(\beta)$, ($h = 6$ mm, $l = 34$ mm).
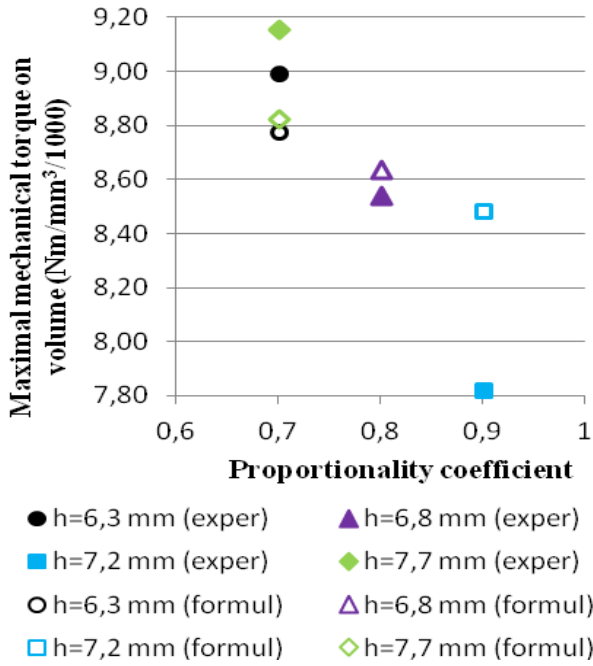
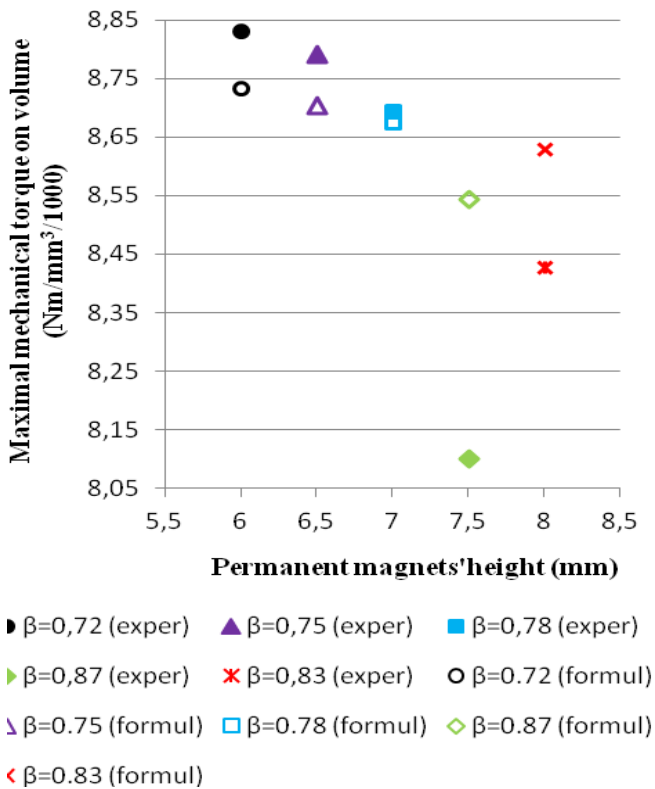Fig. 11. Test variants for varied PM height *h* (*l* = 30 mm).

Fig. 12. Test variants for varied *β* (*l* = 30 mm).

Variant	Proportionality coefficient β	PM's height h (mm)	M_{max}/V (Nm/mm³/1000)		Offset (%)
			Experimental	Formula	
10.	0.73	6.4	8.871	8.733	1.6
11.	0.77	6.7	8.722	8.680	0.5
12.	0.82	7.3	8.468	8.621	1.8
13.	0.86	7.6	8.176	8.565	4.8

The previous analysis showed that for proportionality coefficient *β* = 0.9 the formula gives more diverse values of goal function comparing to experimental data. The same can be seen in Fig. 11, where by standard coefficient *β* = 0.9 for special PM height (*h* = 7.2 mm) is the largest offset. The same situation is seen (Fig. 12), when the coefficient *β* is changed in interval from 0.8 to 0.9 (*β* = 0.83, *β* = 0.87), especially, when the *β* value is closer to 0.9.

For test variants, where variables are varied (coefficient *β* and PM height *h*), the experimental data and the results from formula are given in Table I.

For the main function the results of first and second test variants are given accordingly in Fig. 11 and Fig. 12. The filled markers are for experimental data ("exper") and the markers without fill are for formula given results ("formul").

The offsets of formula results comparing to experimental data are lower than five per cent, such obtained results are acceptable and thus this formula can be used. Though for the proportionality coefficient *β* = 0.9 there must be made additional research to decide, if this is not an exceptional case and another formula should be used.

For the control parameter – yoke's height h_{yoke} – additional variants with the results are given in Table II.

Variant	Proportionality coefficient β	PM's height h (mm)	h_{yoke} (mm)		Offset (%)
			Experimental	Formula	
1.	0.7	6.3	14.65	14.20	3.1
2.	0.8	6.8	17.43	14.81	15.0
3.	0.9	7.2	20.22	15.39	23.9
4.	0.7	7.7	16.32	14.64	10.3
5.	0.72	6	14.70	14.20	3.4
6	0.75	6.5	15.95	14.49	9.2
7.	0.78	7	17.26	14.78	14.3
8.	0.87	7.5	19.99	15.35	23.2
9.	0.83	8	19.77	15.33	22.4
10.	0.73	6.4	15.40	14.37	6.7
11.	0.77	6.7	16.64	14.64	12.0
12.	0.82	7.3	18.56	15.06	18.8
13.	0.86	7.6	19.90	15.34	22.9

TABLE III

MAXIMAL FLUX DENSITY FOR DIFFERENT VARIABLES

Variant	$B_{max\,in}$ (T)		$B_{max\,out}$ (T)	
	experimental	formula	experimental	formula
1.	1.54	1.75	1.29	1.23
2.	1.70	1.81	1.22	1.27
3.	1.94	1.88	1.43	1.31
4.	1.65	1.77	1.24	1.29
5.	1.61	1.75	1.26	1.22
6	1.64	1.78	1.31	1.25
7.	1.78	1.80	1.30	1.27
8.	1.96	1.87	1.49	1.31
9.	1.94	1.85	1.45	1.33
10.	1.72	1.76	1.26	1.24
11.	1.72	1.79	1.27	1.26
12.	1.90	1.83	1.42	1.29
13.	1.87	1.86	1.45	1.31

For four additional variants the offset is under 10%, so the obtained results are not acceptable and further research for yoke's height as control parameter should be made.

For other two control parameters (maximal magnetic flux density in inner and outer half coupling) the results from additional variants are given in Table III. (The values of coefficient β and PM height h are not showed because they are the same as in Table II).

For both maximal magnetic flux densities one additional variant existed where the offset is more than 10%. For maximal magnetic flux density in inner half coupling the average offset is 5.3%, similarly for maximal magnetic flux density in outer half coupling - 5.6%. The results are acceptable, but during optimization such situations must be avoided (bolded variant for $B_{max\ in}$, Table III), when the formula gives higher maximal magnetic flux density than in the experiment, when the controlled value is higher than the necessary. In this case in reality the maximal magnetic flux density would be $B_{max\ in}$ = 1.70 T, but in optimization the formula result $B_{max\ in}$ = 1.81 T would be used. As the control value for $B_{max\ in}$ is 1.80, then, even if it was for the M_{max}/V optimum, this optimum would be abandoned, because 1.81 > 1.80 (T).

V. CONCLUSIONS

From the obtained data it can be concluded, that the highest value of goal function (maximal mechanical torque on volume) is at coefficient equal to β = 0.7, at higher permanent magnets (h = 8 mm) and, of course, at the longest axial length l. These three variables are independent for researching the value of maximal torque, but, when the function for M_{max}/V is

researched, the axial length l is not independent, because it is included in the volume calculation.

The program recognises a situation, that axial length is not an independent variable, and synthesizes the formulas only with two independent variables.

Six formulas were synthesized during the research of the function $M_{max}=f(h, \beta)$ (when two independent variables), from which the one that describes the relevance $M_{max}=f(h, \beta)$ very well was chosen.

Formulas should be researched and synthesized for all control parameters that fit better the experimental results, especially for the yoke's height. For maximal magnetic flux densities in inner and outer half couplings the results are mathematically acceptable. It is important to avoid situations, when the formula gives higher value than it is taken experimentally, especially when the values are: $B_{max\ in} \leq 1.8$ T, $B_{max\ out} \leq 1.6$ T.

The research and formula synthesis must be continued for all pole pair numbers (p from 1 to 10), because in this work only the case when pole pair number is equal 3 was researched.

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