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Sampled-Data Nonlinear Control of ECP-730 Magnetic Levitation System

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Abstract **– This paper presents the idea of implementing various techniques related to sampled-data control for magnetic levitation systems. The control laws are designed to track time-varying signals and employ the feedback linearization technique based on the approximate discrete-time model. State feedback control is introduced with the gains adjusted via the pole placement method. A positional form proportional-integral-derivative (PID) control uses the trapezoidal summation for the integral term and the backward difference method for the derivative term. An input-output linearization feedback control is the most promising one, which incorporates the integrator in addition to the position error and velocity error. The integral action involves the manipulation of regulation error and reduces it with time to improve performance. Finally, controllers were tested in real time for practical demonstration along with a comparison for comprehensive analysis.**

Keywords **– Approximate discrete-time model, input-output linearization feedback control, magnetic levitation system, sampled-data control.**

I. Introduction

Educational control products (ECP) devise the system of magnetic levitation known as ECP Model-730 and provide an excellent opportunity for the scholars to implement the fundamental and advanced level of research work. The research may include the exposition from the areas of linear control, nonlinear control and robust control, etc. The model can be arranged in a variety of ways like SISO, SIMO, or MIMO according to the requirement of work.

PID is extensively used in industrial control applications. The three terms “proportional”, “integral”, and “derivative” entirely perform different functions. At the very first, the difference between the desired outcome and reference input is determined. It is called an error. Then the proportional gain is multiplied by the error term; the integral term computes the integral of error and the last one calculates the derivative of the error. Adding up all the three values forms the control law [1]. The PID controller in [2] tackles the problem of unstable SISO configuration of magnetic levitation system in the frequency domain, carrying out an improvement in the Neimark D-partition method that enhances the results by meeting the specifications of desired phase margin and stability. The dual LQG-PID methodology [3] accomplishes the task of stabilization and regulation of the ion ball levitation system. The PID is employed for the estimation of the signal in the presence of exogenous inputs for stabilization. An LQG-based controller levitates the ball in the vertical direction and manoeuvres the electromagnetic current in such a way that ultimately results in the reduction of error and destabilization. The reliability of the controller depends on the accuracy of the system model. A nonlinear feedback controller for the iron ball electromagnetic suspension system (EMS) is established in [4]. It utilises the approach of exact linearization with a distinguishing feature of robustness against the mass parameter uncertainty. In [5], a mathematical technique aimed at linearization via feedback to address the tracking problem for magnetic levitation systems manufactured by ECP is proposed. Furthermore, an output feedback control accounts for a MIMO setup that enabled magnets to track reference signals from the desired set point.

Sliding mode control (SMC) is robust because it can effectively compensate for the disturbance. The efficacy of sliding mode control over feedback linearized controllers specifically designed for voltage-controlled magnetic levitation systems is verified in [6]. A problem with adaptive tracking control with the application to the MagLev system is discussed in [7]. The dilemma is resolved by converting the non-OFEP nonlinear systems into OFEP, through PFC, i.e., parallel feed-forward compensator which makes the system follow the conditions imposed by OEFP.

This paper aims to study sampled-data controllers, namely state feedback, PID, and input-output linearization controller to achieve better tracking performance for ECP Model-730. In Section II, the description of the mathematical model is formulated. Section III contributes toward the design of control schemes. Section IV deals with the experimental results and a compact comparison for evaluation. Section V concisely states the conclusion.

II. System Model

A. Continuous-Time Nonlinear System

This section provides deep insight into the mathematical model of ECP Model-730 in the continuous-time domain. The configuration of the SISO ECP Model-730 magnetic levitation

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system as shown in Figs. 1 and 2 can be expressed in the mathematical form by (1):

$$m\ddot{y} = F_{m1} - c\dot{y} - F_{m2} - mg. \tag{1}$$

Fig. 1. ECP-730 MagLev apparatus.

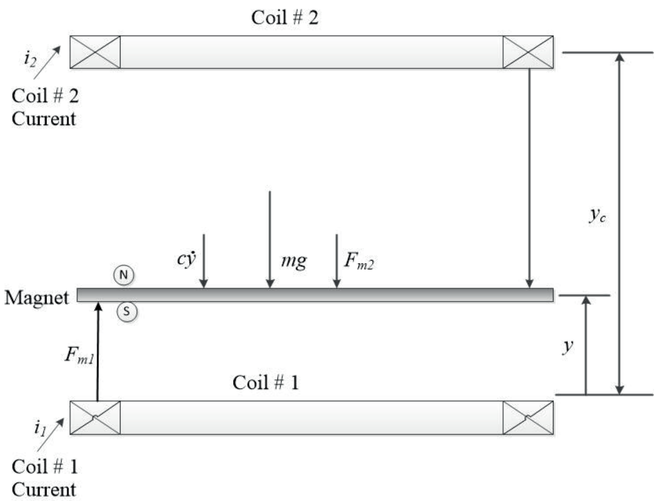

Fig. 2. SISO free body diagram of ECP-730 MagLev apparatus.

In (1), mass, gravitational constant, and friction coefficient characterised by m, g, and c that correspond to the numerical values of 0.121, 9.81, and 1.1, respectively. Moreover, a and b are the constants whose value comes out to be 1.4 and 6.2, which can be verified either by numerical modelling or by empirical methods. The attractive and repulsive forces acting upon the magnetic plate due to the upper coil and lower coil are described by F_{m2} and F_{m1}.

$$F_{m1} = \frac{i_1}{a(y+b)^4}. \tag{2}$$

$$F_{m2} = \frac{i_2}{a(y_c - y + b)^4}. \tag{3}$$

The current for the upper and lower coil is specified by i_1 and i_2. Total displacement is given by y_c, whereas y shows the distance covered by the plate. The input to coil 2 is identical to zero since the configuration of the system contains only one input, so the term $turns$ into zero. Hence, the final equation becomes

$$m\ddot{y} = F_{m1} - c\dot{y} - mg. \tag{4}$$

The nonlinear plant dynamics are expressed as

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -g - \left(\frac{c}{m}\right) x_2 + \frac{u}{ma(x_1 + b)^4}, \tag{5}$$

$$y = x_1$$

where $(x_1, x_2) \in R^2$ shows state, $u \in R$ indicates control law, and $y \in R$ depicts the plant output. The state x_1 represents the position and the state x_2 describes velocity of the magnetic plate.

B. Approximate Discrete-Time Model

The approximate DT model in (6) obtained via the Euler method represents the nonlinear plant model as

$$x_{1a}[k+1] = x_{1a}[k] + Tx_{2a}[k]$$

$$x_{2a}[k+1] = x_{2a}[k] + T \tag{6}$$

$$\left[-g - \left(\frac{c}{m}\right) x_{2a}[k] + \frac{u[k]}{ma(x_{1a}[k] + b)^4}\right].$$

The subscripts 'a' and '*T*' symbolize the approximate state and the sampling time.

III. Discrete-Time Control

This section covers the design of a nonlinear controller based on the approximate discretized system model (6). There are three different techniques adopted for stabilization and tracking. State feedback control employs feedback gains. PID manipulates the error and the gains are adjusted accordingly. Input-output linearization controller includes the higher-order derivatives with a much more complex and sophisticated design.

A. State Feedback Control

The control objective of discrete-time state feedback is to follow the reference trajectory. By using the Euler method the approximate discrete-time model of (5) is

$$x_a[k] = A_{T,d} x_a[\mathrm{k}] +$$

$$B_{T,d}\left[\frac{u[k]}{ma(x_{1a}[k] + b)^4} - g - \left(\frac{c}{m}\right) x_{2a}[k]\right]. \tag{7}$$

The matrices $A_{T,d}$ and $B_{T,d}$ are comprised of the following mathematical expressions:

$$A_{T,d} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, \; B_{T,d} = \begin{bmatrix} 0 \\ T \end{bmatrix}. \tag{8}$$

The vector for feedback gains

$$K_T = [K_1 \quad K_2], \tag{9}$$

$$A_{T,d} - B_{T,d}K_T = \begin{bmatrix} 1 & T \\ -TK_1 & 1 - TK_2 \end{bmatrix}. \tag{10}$$

The discrete-time state feedback linearizing control law is

$$u[k] = ma(x_{1\mathrm{a}}[k] + b)^4$$

$$\left[g\left(\frac{c}{m}\right)x_{2\mathrm{a}}[k] - k_1 x_{1\mathrm{a}}[k] - k_2 x_{2\mathrm{a}}[k]\right]. \tag{11}$$

The Eigen-values or poles are placed at $Q = [1 \quad 0.99735]$ with the values assigned to the vector $K_T = [-1 \quad -0.00645]$.

B. PID Control

PID means proportional integral derivative controller. There are two types of digital PID control schemes, i.e., positional form PID and velocity form PID. Positional form PID is derived by utilising Euler discretization. The derivative term and the integral term are approximated via the forward difference Euler method and trapezoidal summation. The error e relates reference 'r' and the output 'x_1' in discretized form as

$$e[k] = r[k] - x_{1\mathrm{a}}[k]. \tag{12}$$

The discrete-time linearizing control law is

$$u[k] = ma(x_{1\mathrm{a}}[k] + b)^4$$

$$\left\{ \begin{matrix} K_P e[k] + K_I \frac{T}{T_i} \sum_{h=1}^{k} \frac{e[h-1]+e[h]}{2} + \\ K_D \frac{T_d}{T}\left[e[k] - e[k-1]\right] + g + \frac{c}{m} x_{2\mathrm{a}}[k] \end{matrix} \right\}. \tag{13}$$

The gains for PID are $[k_P \quad k_D \quad k_I] = [4 \quad 0.0645 \quad 30]$, while the poles are positioned at $Q = [0.9888 \quad 0.9531 \pm 0.0822i]$.

C. Input-Output Linearization Feedback Control

An input-output feedback linearizing control was specifically devised for tracking. The relative degree of the magnetic levitation system (5) comes out to be 2. The design includes the position error and velocity error. Moreover, the integrator computes the integral of error, providing an extra degree of freedom, and enhancing the performance.

The reference terms and error co-ordinates in discrete time

$$R = \begin{bmatrix} r[k] \\ \frac{r[k+1]-r[k]}{T} \end{bmatrix},$$

$$\begin{bmatrix} e_1[k] \\ e_2[k] \end{bmatrix} = \begin{bmatrix} x_{1\mathrm{a}}[k] - r[k] \\ x_{2\mathrm{a}}[k] - \frac{r[k+1]-r[k]}{T} \end{bmatrix}, \tag{14}$$

$$\begin{bmatrix} e_1[k] \\ e_2[k] \end{bmatrix} = \begin{bmatrix} x_{1\mathrm{a}}[k] - r[k] \\ \frac{x_{1a}[k+1]-x_{1a}[k]}{T} - \frac{r[k+1]-r[k]}{T} \end{bmatrix}. \tag{15}$$

The derivative of the error term (15) becomes

$$\begin{bmatrix} e_1[k+1] \\ e_2[k+1] \end{bmatrix} = \tag{16}$$

$$\begin{bmatrix} e_1[k] + Te_2[k] \\ e_2[k] + T\left(\begin{matrix} -g - \left(\frac{c}{m}\right)x_{2\mathrm{a}}[k] + \frac{u[k]}{ma(x_{1\mathrm{a}}[k]+b)^4} - \\ \frac{r[k+2]-2r[k+1]+r[k]}{T^2} \end{matrix}\right) \end{bmatrix}.$$

The linearizing state feedback control law

$$u = ma(x_{1\mathrm{a}}[k] + b)^4 \Big\{ -k_1\, e_1[k] - k_2\, e_2[k] - k_3 * T * \sum_{h=1}^{k} \frac{e_1[h-1]+e_1[h]}{2} + g + \left(\frac{c}{m}\right)x_{2\mathrm{a}}[k] + \frac{r[k+2]-2r[k+1]+r[k]}{T^2} \Big\}, \tag{17}$$

where the linear feedback control contains the integral of the error represented by 'σ'. It can be defined as

$$V = -\begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix} \begin{bmatrix} e_1[k] \\ e_2[k] \\ \sigma \end{bmatrix}, \; \sigma = T * \sum_{h=1}^{k} \frac{e_1[h-1]+e_1[h]}{2}.$$

The values of the feedback with the poles located at $Q = [0.9888 \quad 0.9531 \pm 0.0822i]$.

IV. Experimental Results

This section deals with the comparison of input-output linearization feedback control, PID control, and state feedback control. All the control schemes are designed via Euler discretization. The performance is measured against a sinusoid trajectory with an amplitude of 2 cm, frequency of 0.5 Hz, and the sampling time $T_s = 0.002652$ s.

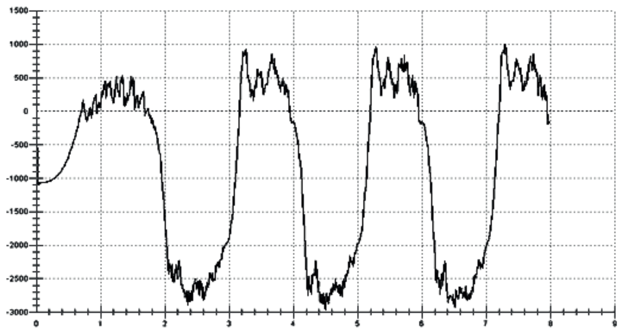

Fig. 3. Tracking error of state feedback control for sinusoid reference.

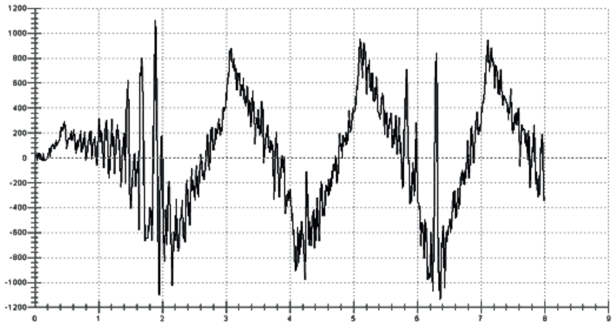

Fig. 4. Tracking error of PID control for sinusoid reference.

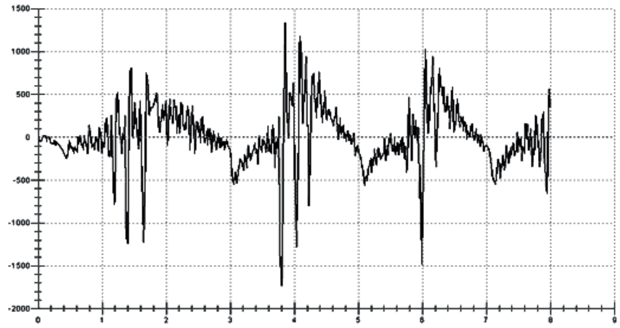

Fig. 5. Tracking error of input-output linearization feedback control for sinusoid reference.

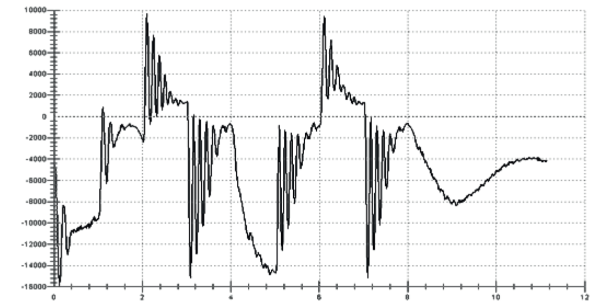

Fig. 6. Tracking error of input-output linearization feedback control for sinusoid reference with step disturbance.

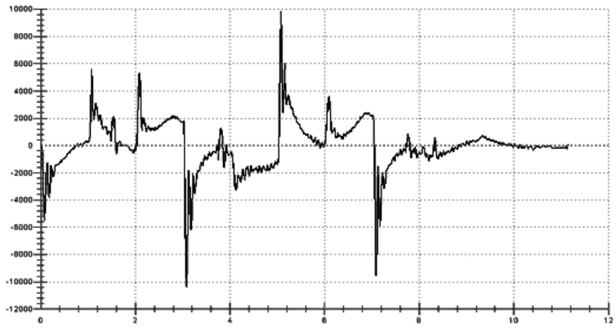

Fig. 7. Tracking error of PID control for sinusoid reference with step disturbance.

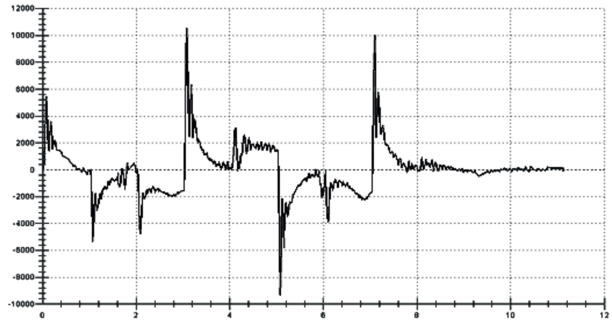

Fig. 8. Tracking error of input-output linearization feedback control for sinusoid reference with step disturbance.

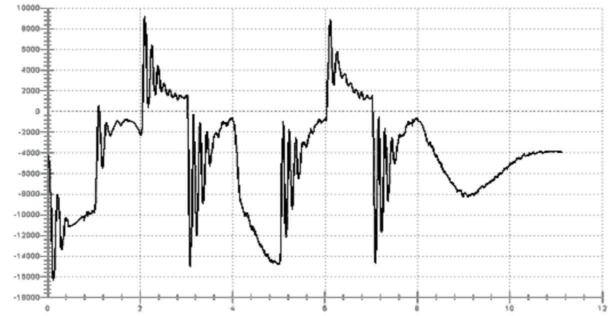

Fig. 9. Tracking error of state feedback control for sinusoid reference with step disturbance and perturbed mass.

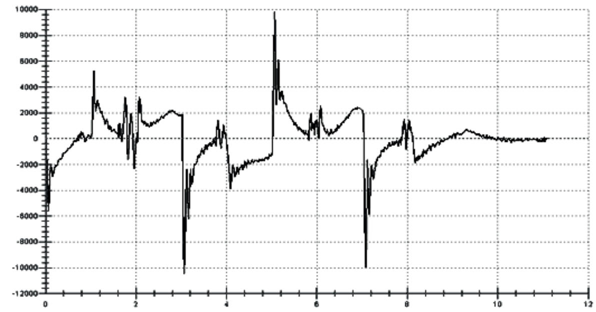

Fig. 10. Tracking error of PID control for sinusoid reference with step disturbance and perturbed mass.

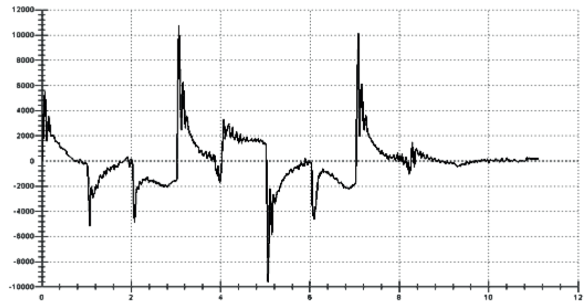

Fig. 11. Tracking error of input-output linearization feedback control for sinusoid reference with step disturbance and perturbed mass.

The experimental results are shown in Figs. 3–5 for the sinusoid reference. When a step disturbance is added to the input, with a sinusoid trajectory to follow, the results obtained are displayed in Figs. 6–8. When the mass is perturbed, in addition to the step disturbance and a sinusoid reference, the results can be depicted in Figs. 9–11. The vertical axis contains the value measured in counts and the horizontal axis shows the time in seconds. The parameters adopted for the performance analysis comprised mean and standard deviation. The values of mean and standard deviation against error plots for the designed control schemes are provided in Tables I–III.

TABLE I

COMPARISON OF REAL TIME IMPLEMENTATION TRACKING RESULTS FOR SINUSOID REFERENCE

Control Scheme	Mean	Standard Deviation
State Feedback	−897.2027	1316.8
PID	24.6777	434.2103
Input-Output Feedback Linearization	−9.13030	357.9700

TABLE II

COMPARISON OF REAL TIME IMPLEMENTATION TRACKING RESULTS FOR SINUSOID REFERENCE WITH DISTURBANCE

Control Scheme	Mean	Standard Deviation
State Feedback	1022.4	4950.1
PID	−3.0191	1809.5
Input-Output Feedback Linearization	0.5202	1794.4

TABLE III

COMPARISON OF REAL TIME IMPLEMENTATION TRACKING RESULTS FOR SINUSOID REFERENCE WITH DISTURBANCE AND PERTURBATION

Control Scheme	Mean	Standard Deviation
State Feedback	1100.4	4926.4
PID	−3.6181	1831.0
Input-Output Feedback Linearization	1.6508	1827.9

V. CONCLUSION

In this paper, the performance of sampled-data controls, i.e., state feedback control, PID control, and input-output feedback linearization control for magnetic levitation system, is analysed. The Euler method is used for the discretization of the system model. Mean and standard deviation are the tools employed for the comparison of control schemes. From Table I, it can be concluded that the input-output feedback linearization controller is the most efficient one, as it owns the lowest values of mean and standard deviation. Furthermore, the PID controller yields better results than the state feedback controller. Afterward, the tracking of controllers is observed in the presence of step disturbance for a sinusoid reference. The results are summarised in Table II. Table III contains the results for perturbed mass and step disturbance for a sinusoid trajectory. The pattern of performance remains the same, i.e., input-output feedback linearization controller performs the best, following the PID and state feedback controller.

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