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Middle is the Invariant Unique Optimal Time Moment in Exponentially-Concave-Reward 1-Bullet Progressive Silent Duel

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Abstract **– The finite 1-bullet progressive silent duel is considered, in which each of the two duellists shoots with exponentially-concave reward. The duel models a system of one-decision-making competition between two identical intelligent competitors (duellists) through a quantized time span, in which the duellist benefits from shooting as late as possible but only by acting first. The duel is silenced because the duellist does not learn about the action of the other duellist until the very end moment of the duel. Thus, the duel is a symmetric matrix game whose optimal value is 0, and the set of optimal strategies is the same for both duellists, regardless of the duel size and how time is quantized. The duel time quantization is such that time progresses by the geometrical progression pattern, according to which every following time moment is the partial sum of the respective geometric series. In this duel, the duellist has the invariant unique optimal time moment, which is the duel middle, regardless of the number of time moments. Usually, the system manager benefits namely from such a solution, by which the manager latently forces both competitors to act at the invariant optimal moment, maintaining the system more stable and controllable.**

Keywords **– 1-bullet silent duel, exponentially-concave reward, matrix game, optimal time moment, time progression.**

I. One-Decision-Making Competition

One-decision-making competitions emerge in environments with limited or restricted resources to use, where the time interval to act is squeezed to a standardised unit and the competition ends following this unit, during which competitors must take an action [1], [2]. The competitor benefits from acting as late as possible but only by acting first [3], [4]. Typical examples include decentralised financial systems (miners or validators compete in a race to timely propose the next block) [5], [6], advertising (entrepreneurs compete for timely posting ads) [3], [7], [8], jurisprudence (in court trials, prosecutors and defenders compete for timely exhibiting their evidences), general publicising (civil and government authorities compete for timely exposing facts or events that may have dramatic impact on society and power layout, respectively) [9]. Many of such examples involve just two sides. Thus, they are modelled by finite 1-bullet silent duels as a symmetric matrix game [4], [10], [11]

$$\left\langle X_N, Y_N, \mathbf{U}_N \right\rangle = \left\langle \left\{x_i\right\}_{i=1}^{N}, \left\{y_j\right\}_{j=1}^{N}, \mathbf{U}_N \right\rangle \quad \text{by } X_N = Y_N = T_N \tag{1}$$

with duel time span

$$T_N = \left\{t_q\right\}_{q=1}^{N} \subset [0;1] \quad \text{by } t_q < t_{q+1} \;\; \forall\, q = \overline{1, N-1} \quad \text{and } t_1 = 0,\; t_N = 1 \text{ for } N \in \mathbb{N} \setminus \{1, 2\} \tag{2}$$

and a skew-symmetric payoff matrix

$$\mathbf{U}_N = \left[u_{ij}\right]_{N\times N} = \left[-u_{ji}\right]_{N\times N} = -\mathbf{U}_N^{\mathrm{T}} \tag{3}$$

of the first duellist's rewards [12]. Duel (1) with (2) and (3) is silenced because the duellist does not learn about the action of the other duellist until the duel very end moment $t_N = 1$ [2], [13], [14].

The solution of duel (1) with (2) and (3) is determined by how internal moments $\left\{t_q\right\}_{q=2}^{N-1}$ are assigned within interval $(0;1)$ and how matrix (3) is structured [3], [4], [10], [12]. Owing to the matrix game symmetry, the set of optimal strategies is the same for both duellists, and the optimal value of the game is 0 [12], [15]. In addition, optimal pure strategies are more preferable than optimal mixed strategies because practical realisation of the optimality relies on fewer duel repetitions due to changing conditions of the environment.

The structure of the finite 1-bullet progressive silent duel yields a set of practically valuable insights for the design of timing-critical mechanisms both in smart-contract systems and communication engineering [16], [17]. Many distributed digital environments require autonomous agents to make a single irreversible decision within a quantized time grid, while being unable to observe the actions of their competitors until the protocol's end; this pattern arises not only in blockchain-based commitment schemes but also in media-access control (MAC)

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[18], contention-based protocols [16], distributed scheduling [11], [13], random-access competition [1], [2], [4], [5], [10], spectrum sharing [19], [20], and time-triggered communication frameworks [21].

In such settings, agents face a dual timing incentive analogous to that in the progressive silent duel: acting later increases informational advantage or payoff, but acting first is required to secure the benefit [22], [23]. Whenever action opportunities are discretized (slots, epochs, frames, scheduled windows) and opponents' decisions remain hidden, the strategic environment becomes structurally equivalent to a one-bullet duel on a quantized timeline [24], [25].

The duel's central theoretical implication is: the rational competitors converge to one invariant optimal action moment regardless of the number or density of time checkpoints. This can be operationalised as an engineering design principle. A protocol designer can embed an exponentially concave reward curve and a geometrically progressing time grid into a smart contract or a communication procedure, thereby guiding participants toward a predictable equilibrium timing without explicit coordination [26], [27]. This reduces premature contention, mitigates undesirable race conditions, and stabilises the system by creating a latent coordination point [28], [29].

In communication engineering, similar timing-equilibrium effects are known to improve collision avoidance, reduce network backoff variance, and enhance determinism in systems such as slotted ALOHA extensions [30], contention-based sensor networks, reservation protocols, and distributed resource-allocation schemes [31], [32]. By aligning incentive-compatible timing with the geometric time progression, system architects can reduce overhead in state verification, lower energy waste caused by repeated contention attempts, and improve the predictability of access patterns [33], [34].

In smart-contract design, the existence of a deterministic equilibrium timing simplifies the contract's internal logic, since the designer can engineer the environment so that symmetric rational agents voluntarily synchronise on the desired action epoch [17], [26]. This reduces incentive-driven timing distortions, minimises digital (blockchain) gas inefficiencies caused by premature or redundant state updates, and enhances operational robustness by implicitly coordinating agent behaviour. More broadly, the duel demonstrates how time-dependent payoff curves can serve as latent coordination mechanisms under strict information asymmetry, which is a fundamental challenge in both blockchain consensus engineering and distributed communication systems [6], [35], [36].

II. Reward Exponential Rate with Saturation

Duel (1) with (2) and (3) has N successive time moments of possible shooting (acting) [3], [4], [12], [14]. Assigning internal moments $\{t_q\}_{q=2}^{N-1}$ of possible shooting must regard the growing tension, responsibility, and plausible anxiety as the duel progresses. Therefore, as the duellist approaches the end moment $t_N = 1$, the space between consecutive moments t_q and t_{q+1}, $q=\overline{1, N-1}$, should not shorten [4], [10], [11]:

$$t_q - t_{q-1} \geqslant t_{q+1} - t_q \quad \forall\, q = \overline{2, N-1} \text{ for } N \in \mathbb{N} \setminus \{1, 2\}. \tag{4}$$

Nevertheless, the density of internal moments $\{t_q\}_{q=2}^{N-1}$ must gradually grow as the duellist approaches to the duel end [13], [37]. One of the patterns of such growth is the geometrical progression, according to which every following moment is the partial sum of the respective geometric series [4], [10], [11], [38]:

$$t_q = \sum_{l=1}^{q-1} 2^{-l} = \frac{2^{q-1}-1}{2^{q-1}} \text{ for } q = \overline{2, N-1}.$$

Then, game (1) with

$$T_N = \left\{0, \left\{\frac{2^{q-1}-1}{2^{q-1}}\right\}_{q=2}^{N-1}, 1\right\} \text{ for } N \in \mathbb{N} \setminus \{1, 2\} \tag{5}$$

and (3) is a finite 1-bullet progressive silent duel whose time schedule obeys (4) as

$$t_q - t_{q-1} > t_{q+1} - t_q \quad \forall\, q = \overline{2, N-2} \text{ for } N \in \mathbb{N} \setminus \{1, 2, 3\} \tag{6}$$

by

$$t_{N-1} - t_{N-2} = t_N - t_{N-1} = \frac{1}{2^{N-2}} \text{ for } N \in \mathbb{N} \setminus \{1, 2\}. \tag{7}$$

Thus, according to (7), the last two duel time subintervals are of the same length. It is true for the 3×3 duel as well, where set (5) is $T_3 = \left\{0, \frac{1}{2}, 1\right\}$ still obeying condition (7), which is a partial case of requirement (4). Condition (6) emerges from bigger duels.

Apart from the duel time schedule, the resulting payoff of the first duellist in finite 1-bullet silent duels is

$$u_{ij} = g(x_i) - g(y_j) + g(x_i)\, g(y_j)\, \mathrm{sign}(y_j - x_i) \text{ for } i = \overline{1, N} \text{ and } j = \overline{1, N} \tag{8}$$

by some discrete reward functions $g(x_i)$ and $g(y_j)$ of the first and second duellists, respectively, where

$$g(t_1) = g(0) = 0 \text{ and } g(t_N) = g(1) = 1. \tag{9}$$

Generally speaking, discrete reward function $g(t_q)$ must be non-decreasing [2], [3], [12], [13]. However, usually it is an increasing function and it is appropriate to consider an exponentially-increasing reward function with saturation:

$$g(t_q) = ae^{-t_q} + b \text{ by } a \in \mathbb{R} \setminus \{0\},\ b \in \mathbb{R}. \tag{10}$$

As function (10) of discrete variable t_q must obey requirements (9), then

$$g(t_0) = g(0) = a + b = 0\,,$$

$$g(t_N) = g(1) = ae^{-1} + b = 1\,,$$

whence

$$b = -a = 1 - ae^{-1}\,,$$

$$a\left(e^{-1} - 1\right) = 1\,,$$

and

$$a = \frac{1}{e^{-1} - 1} = \frac{e}{1 - e}\,,\ b = \frac{e}{e - 1}\,. \qquad (11)$$

Upon plugging (11) into (10) function $g(t_q)$ becomes an exponentially-concave-reward function:

$$g(t_q) = \frac{e \cdot e^{-t_q}}{1 - e} - \frac{e}{1 - e} = \frac{e \cdot \left(e^{-t_q} - 1\right)}{1 - e}\,. \qquad (12)$$

Then, upon plugging (12) into (8), entry u_{ij} of payoff matrix (3) is calculated as follows:

$$u_{ij} = \frac{e \cdot \left(e^{-x_i} - 1\right)}{1 - e} - \frac{e \cdot \left(e^{-y_j} - 1\right)}{1 - e} + \\ + \frac{e \cdot \left(e^{-x_i} - 1\right)}{1 - e} \cdot \frac{e \cdot \left(e^{-y_j} - 1\right)}{1 - e} \cdot \operatorname{sign}\left(y_j - x_i\right) = \\ = \frac{e \cdot \left(e^{-x_i} - e^{-y_j}\right)}{1 - e} + \frac{e^2 \cdot \left(e^{-x_i} - 1\right)\left(e^{-y_j} - 1\right)}{(1 - e)^2} \cdot \operatorname{sign}\left(y_j - x_i\right)$$

$$\text{for } i = \overline{1, N} \text{ and } j = \overline{1, N}\,. \qquad (13)$$

III. Objective

The objective is to determine the set of optimal time moments $\Theta(N) \subset T_N$ for exponentially-concave-reward 1-bullet silent duel (1) with (5) and (3) as (13). Herein, it ought to be noted that

$$u_{1j} = \frac{e \cdot \left(1 - e^{-y_j}\right)}{1 - e} + \frac{e^2 \cdot (1 - 1) \cdot \left(e^{-y_j} - 1\right)}{(1 - e)^2} = \\ = \frac{e \cdot \left(1 - e^{-y_j}\right)}{1 - e} < 0 \quad \forall\, j = \overline{2, N} \qquad (14)$$

due to $1 - e < 0$ and $e^{-y_j} < 1 \ \ \forall\, y_j > 0$. Inequality (14) means that the duel beginning moment $t_1 = 0$ is never optimal, regardless of the number of time moments:

$$t_1 \notin \Theta(N) \quad \forall\, N \in \mathbb{N} \setminus \{1, 2\}\,. \qquad (15)$$

IV. Duel Middle

First of all, it is about ascertaining a distinctive property of the payoff matrix (3). This property will help determining set $\Theta(N)$.

Theorem 1. Entry u_{nj} by (13), considered as a discrete function of index $j = \overline{1, n-1}$ by $n \in \left\{\overline{2, N}\right\}$, strictly decreases as index j is increased. Entry u_{nj} by (13), considered as a discrete function of index $j = \overline{n+1, N}$ by $n \in \left\{\overline{2, N-1}\right\}$, strictly decreases as index j is increased.

Proof. Plugging $i = n$ into (13) for $n \in \left\{\overline{2, N}\right\}$, entry

$$u_{nj} = \frac{e \cdot \left(e^{-x_n} - 1\right)}{1 - e} - \frac{e \cdot \left(e^{-y_j} - 1\right)}{1 - e} - \\ - \frac{e \cdot \left(e^{-x_n} - 1\right)}{1 - e} \cdot \frac{e \cdot \left(e^{-y_j} - 1\right)}{1 - e} = \\ = \frac{e \cdot \left(e^{-x_n} - 1\right)}{1 - e} - \left(1 + \frac{e \cdot \left(e^{-x_n} - 1\right)}{1 - e}\right) \cdot \frac{e \cdot \left(e^{-y_j} - 1\right)}{1 - e}$$

$$\text{for } j = \overline{1, n-1} \text{ at } n \in \left\{\overline{2, N}\right\}. \qquad (16)$$

Due to $e^{-x_n} < 1$ and $e^{-y_j} \leqslant 1$ by $x_n > 0$ and $y_j \geqslant 0$, respectively, and

$$-\left(1 + \frac{e \cdot \left(e^{-x_n} - 1\right)}{1 - e}\right) \cdot \frac{e}{1 - e} > 0\,,$$

entry (16) decreases because exponent e^{-y_j} decreases as index j is increased off 1 up to $n - 1$.

Plugging $i = n$ into (13) for $n \in \left\{\overline{2, N-1}\right\}$, entry

$$u_{nj} = \frac{e \cdot \left(e^{-x_n} - 1\right)}{1 - e} - \frac{e \cdot \left(e^{-y_j} - 1\right)}{1 - e} + \\ + \frac{e \cdot \left(e^{-x_n} - 1\right)}{1 - e} \cdot \frac{e \cdot \left(e^{-y_j} - 1\right)}{1 - e} = \\ = \frac{e \cdot \left(e^{-x_n} - 1\right)}{1 - e} - \left(1 - \frac{e \cdot \left(e^{-x_n} - 1\right)}{1 - e}\right) \cdot \frac{e \cdot \left(e^{-y_j} - 1\right)}{1 - e}$$

$$\text{for } j = \overline{n+1, N} \text{ at } n \in \left\{\overline{2, N-1}\right\}. \qquad (17)$$

Inasmuch as

$$e^{-1} < e^{-x_n} < 1\,,\ 1 < e \cdot e^{-x_n}\,,\ e^{-y_j} \leqslant 1\,,$$

and

$$-\left(1 - \frac{e \cdot \left(e^{-x_n} - 1\right)}{1 - e}\right) \cdot \frac{e}{1 - e} = \left(1 - \frac{e \cdot \left(e^{-x_n} - 1\right)}{1 - e}\right) \cdot \frac{e}{e - 1} > 0$$

due to

$$1-\frac{e\cdot\left(e^{-x_n}-1\right)}{1-e}=\frac{1-e-e\cdot\left(e^{-x_n}-1\right)}{1-e}=\frac{1-e\cdot e^{-x_n}}{1-e}>1,$$

entry (17) decreases due to exponent e^{-y_j} decreases as index j is increased off $n+1$ up to N. □

Hence, **Theorem 1** reads that every row of payoff matrix (3) is sort of decaying towards the main diagonal and forth off the main diagonal. This fundamentally helps prove the following assertion.

Theorem 2. The middle time moment $t_2=\frac{1}{2}$ is the invariant unique optimal time moment in exponentially-concave-reward 1-bullet silent duel (1) with (5) and (3) as (13):

$$\Theta(N)=\{t_2\}=\left\{\frac{1}{2}\right\}\;\forall N\in\mathbb{N}\setminus\{1,2\}. \quad (18)$$

Proof. Time moment $t_2=\frac{1}{2}$ is singly optimal if $u_{21}>0$ and

$$u_{2j}>0\;\;\forall j=\overline{3,N}. \quad (19)$$

Inequality $u_{21}>0$ holds owing to $u_{12}<0$ by (14) and $u_{21}=-u_{12}$ by (3). Owing to **Theorem 1**, function u_{2j} is decreasing with respect to index $j=\overline{3,N}$, and hence inequality (19) is equivalent to inequality

$$u_{2N}=\frac{e\cdot\left(e^{-x_2}-e^{-y_N}\right)}{1-e}+\frac{e^2\cdot\left(e^{-x_2}-1\right)\left(e^{-y_N}-1\right)}{(1-e)^2}=$$
$$=\frac{e\cdot\left(e^{-\frac{1}{2}}-e^{-1}\right)}{1-e}+\frac{e^2\cdot\left(e^{-\frac{1}{2}}-1\right)\left(e^{-1}-1\right)}{(1-e)^2}=$$
$$=\frac{e\cdot\left(e^{-\frac{1}{2}}-e^{-1}\right)}{1-e}+\frac{e^2\cdot\left(e^{-\frac{1}{2}}-1\right)\cdot\frac{1-e}{e}}{(1-e)^2}=$$
$$=\frac{2\sqrt{e}-e-1}{1-e}>0.2,$$

i.e., inequalities (19) hold. □

Thus, owing to **Theorem 2**, the system modelled by the exponentially-concave-reward 1-bullet progressive silent duel is managed more effectively: the system manager must only properly schedule the time moments of possible acting, whereas the competitors themselves decide on acting at the middle time moment (because otherwise a loss is guaranteed). In this way, the competitor is, so to speak, latently forced to act at the invariant optimal moment. This, nevertheless, prevents the system from instabilities and outliers.

V. Comparison to Uniform Time Quantization

Progressive time quantization by (5) is not only for simulating the growing tension, responsibility, and anxiety as the duel progresses. It actually creates the duel solution invariant by (18). Otherwise, if, say, time is quantized uniformly, then [14]

$$T_N=\left\{t_q\right\}_{q=1}^{N}=\left\{\frac{q-1}{N-1}\right\}_{q=1}^{N}\subset[0;1]$$
$$\text{for } N\in\mathbb{N}\setminus\{1,2\} \quad (20)$$

and no duel solution invariant exists for exponentially-concave-reward 1-bullet silent duel (1) with (5) and (3) as (13). In this case, pure strategy solutions exist only when the duellist has three, five, or six time moments to shoot:

$$\Theta(3)=\{t_2\}=\left\{\frac{1}{2}\right\},$$

$$\Theta(5)=\{t_3\}=\left\{\frac{1}{2}\right\},$$

$$\Theta(6)=\{t_3\}=\left\{\frac{2}{5}\right\},$$

whereas

$$\Theta(N)=\varnothing\;\;\forall N\in\mathbb{N}\setminus\{1,2,3,5,6\}.$$

If time progresses by set (20) and, instead of exponentially-concave-reward function (12) for (8), linear-reward function $g(t_q)=\alpha t_q$ is used by $\alpha>0$ [39], set $\Theta(N)$, which now depends on factor α, cannot be made invariant, unless this factor is set to specific values. Thus,

$$\Theta(N)=\{t_N\}=\{1\}$$
$$\forall\alpha\in\left(0;\frac{1}{N-2}\right)\text{ and }\forall N\in\mathbb{N}\setminus\{1,2\},$$

$$\Theta(N)=\left\{t_q\right\}=\left\{\frac{q-1}{N-1}\right\}$$
$$\forall\alpha\in\left(\frac{N-q}{q-1};\frac{N-1}{(q-2)\cdot(q-1)}\right]$$
$$\text{for } N\in\mathbb{N}\setminus\{1,2,3\}\text{ and } q\in\left\{\overline{3,N}\right\},$$

$$\Theta(N)=\{t_2\}=\left\{\frac{1}{N-1}\right\}$$
$$\forall\alpha>N-2\text{ and }\forall N\in\mathbb{N}\setminus\{1,2\}.$$

However, setting linear-reward factor α to those specific values is not always possible in practice. Progressive time quantization by (5) along with exponentially-concave rewarding by (12) are realised easier.

VI. Conclusion

In exponentially-concave-reward 1-bullet progressive silent duel (1) with (5) and (3) as (13), modelling a system of one-decision-making competition between two identical intelligent competitors (duellists) through a quantized time span, the duellist has the single optimal strategy. In this duel, the duellist has the invariant unique optimal time moment, which is the duel middle, regardless of the number of time moments. Usually, the system manager benefits namely from such a solution, by which the manager latently forces both competitors to act at the invariant optimal moment, maintaining the system more stable and controllable.

The obtained result has practical implications for smart-contract based decentralised energy trading. Thus, translating this timing-equilibrium insight into decentralised energy markets, a smart-contract platform can embed time-dependent incentive rules so that prosumers or energy suppliers (competing to sell energy) effectively coordinate on a single submission moment. For example, in a peer-to-peer energy trading smart contract, bids or offers could be accepted only at a predetermined "middle slot" derived from the slot schedule, encouraging all suppliers to submit simultaneously at that slot. This reduces the overhead of continuous bidding, lowers transaction collisions, and simplifies consensus on which offer gets matched first, thereby improving system stability and throughput. Recent studies have demonstrated that smart-contract driven energy trading systems with proper scheduling and automated trade execution achieve enhanced efficiency, reduced transaction cost and lower volatility in multi-participant scenarios [40], [41]. Moreover, the approach helps grid operators and decentralised energy-market designers implement latent coordination without requiring explicit communication or trust, which is critical for scalability and fairness in microgrid or distributed energy resource networks [42].

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