

# Research into Performance Index-Dependent Student Learning Dynamics

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**Abstract** – The problem of effective student learning of a new course within the framework of rational student goal setting is an “eternal” applied problem for the social and technical sciences. Each rational goal setting of the student was formulated in the form of a non-obvious mathematical construct of a nonlinear objective function that determined the minimized functional for the corresponding optimal control problem. Within the framework of the author’s approach to nonlinear modelling of various optimal goal-settings in student study of a new course, 35 optimal control problems for 35 pedagogically-admissible algebraic constructs for minimized functionals were mathematically posed, Optimica-formulated and numerically solved in JModelica-{1.17; 2.14}. As part of further generalization and psychological and pedagogical interpretation of the obtained graphical results of the numerical modelling, the following six strategies for studying a new course by a student were formulated: “Lazy Student” (Strategy A); “Procrastinator” (Strategy B); “Growing Student” (Strategy C); “Steady Student” (Strategy D); “Midterm Hero” or “Halfway Hero” (Strategy E), and “Starting Hero” or “Sprinting Hero” (Strategy F). The six strategies mentioned above for student learning of a new course seem to be a fairly concise summary of the individual educational efforts of both a school student, a university student, and a working professional.

**Keywords** – Computational cybernetics, continuing education, control engineering education, control nonlinearities, industrial psychology, key performance indicator, optimal control.

## I. INTRODUCTION AND MAJOR CHALLENGES

Systematic and purposeful study by a student of a certain new course applies to both the formal education of vocational or higher education applicants who simultaneously and in parallel study several new disciplines and subsequently, upon completion of officially regulated education, apply for a state-standard diploma, and the long-term technical or engineering education of working locksmiths or engineers who already have an official diploma of vocational or higher education and can currently focus their time and resources on the purposeful consistent study of only one specific discipline, and are also largely interested in obtaining an official certificate from the studied target course within the processes of continuous professional development of working specialists [1]–[43].

The educational problem of learning dynamics of the lazy and forgetful student became very popular in the last 45 years and was computationally addressed by the following

researchers: Raggett et al. [1], Bondi [2], Woodside [3], Parlar [4], Klamkin [5], Cheng et al. [6], Lee et al. [7]–[8], [10], Chen [9], Buratto et al. [11], [12], Smith et al. [13], Mehmood [14], Bao et al. [15], Brunetto et al. [16], Goldt et al. [17], Rachim [18], Asanuma et al. [19], Castaldi et al. [20], Chorny et al. [21], Kaffenberger et al. [22], Kooken et al. [23], Teo et al. [24], [25], Lewis [26], Teklu et al. [27], Dominé et al. [28], Qiu [29], Bhih et al. [30], Vergaño [31], Castaldi et al. [32], Loong et al. [33], Papageorgiou et al. [34], Zine et al. [35].

Let us analyse the work by Klamkin (1985) [5]. The differential equation (DESLD) of the dynamics of student learning is Eq. (8) in [5]:  $(d(x_1(t))/dt) = (-c) \times (x_1(t)) + b \times ((u(t))^\gamma)$ , i.e., the rate of change of the student’s knowledge level  $(d(x_1(t))/dt)$  is equal to the negative product of the forgetting factor of the studied material  $(-c)$ , multiplied by the student’s knowledge level  $x_1(t)$  plus the product of the learning efficiency factor  $b$  multiplied by the control signal, raised to the exponent in the form of a limiting factor  $((u(t))^\gamma)$ . The nonlinear objective function in [5] is the minimized functional:  $J = \int(0 \text{ to } T) u(t) dt$  in the form of a definite integral of the control signal  $u(t)$  over the entire time  $T$  of studying the new course [5]. The goal of [5] is to solve the optimal control problem for the differential equation of student dynamics (DESLD) within the framework of minimizing the functional:  $\min(J) = \min(\int(0 \text{ to } T) u(t) dt)$ . The graphical representation of the data in [5] is the schematic graph of the discrete control signal, as shown in Fig. 1 of [5]. Advantages of the work [5]: In the context of calculating the minimum academic work  $J_{\min}$  of a lazy student for uneven study of a new course, as well as by calculating the uniform academic work  $J_{\text{unif}}$  of a student for uniform study of a course, algebraic formulas (10) and (12) were obtained for estimating the minimum value  $J_{\min}$  of the minimized functional  $J$ . Disadvantages of data presentation in the work [5]: there is the absence of detailed calculation graphs for the student’s knowledge level  $x_1(t)$  and for the control signal  $u(t)$ , since the schematic (Fig. 1) in [5] for the discrete value of the control signal does not contain a scale of numerical values along the vertical axis.

As part of the computational implementation of the sociocybernetic approach to nonlinear dynamic modelling of educational processes of studying a new university academic discipline by engineering students, Chorny et al. [21] used the

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methods of automatic control theory, automated electric drive, as well as the fractal theory of fractional  $\mu$ -order derivatives. They performed a numerical system-dynamic solution of an ordinary inhomogeneous differential equation (1), formally similar to Newton's second law, which is a second-order ODE with respect to the variable  $S$ , containing on the left such positive terms as the first time derivative  $S' = (dS/dt)$  and the second time derivative  $S'' = (d(dS/dt)/dt)$ , i.e., the left-hand side of which was written with respect to the first variable  $S$  as a function of time  $S = S(t)$  (Figs. 1–8), denoting the information flow of new educational information successfully learned by the student, and the right-hand side of which contained the second variable  $H$  as a function of time  $H = H(t)$ , denoting the general flow of initial information received within the framework of studying a new academic subject, and the Laplace transform  $H(p)$  of the transition function  $H(t)$  was alternately and sequentially modelled either by a fractal aperiodic link with the image  $H_1(p)$ , containing the fractal derivative of fractional order  $\mu$ , or by a fractal aperiodic link with the image  $H_2(p)$  of order  $1 + \mu$ , or by a more complex link with the image  $H_3(p)$ , defined as the product of the inertial link and the image  $H_1(p)$ , or by a more complex link with the image  $H_4(p)$ , defined as the product of the inertial link and the image  $H_2(p)$ , which led to the need to perform a numerical solution of four corresponding equations describing the student's study of new educational material, with the same left-hand sides, i.e., written relative to the variable  $S$  on the left, and containing different right-hand sides corresponding to the functions  $\{H_1, \dots, H_4\}$  on the right [21]. In the framework of a phenomenological computational analysis of the first 100 days of a student's study of a new academic discipline, Chorny et al. [21] were able to successfully visualize complex transient processes for three coupled curves  $S = S(t)$ . They comprehensively illustrated the nonlinear educational dynamics of the most successful study of a new course by excellent students with a high degree of convergence of all three  $S_1$ -curves (Fig. 6), nonlinear oscillatory processes of average successful mastering of a new course by good students with a greater degree of relative divergence of  $S_2$ -curves (Fig. 7) and nonlinear oscillatory processes of poorly successful mastering of a new course by C-students with the maximum degree of relative divergence of  $S_3$ -curves (Fig. 8). The authors used system-dynamic construction of the fractal structure of the function  $H$ , as well as within the framework of an empirical assessment of the numerical values of the parameters of the differential equation (1) based on statistical processing of the results of pedagogical experiments, conducted by the authors of the work [21].

In the recent paper [26], several nonlinear optimisation problems of student productivity were considered as Examples 1–4, Remarks 1–4 and Figs. 1–4 within the framework of applying computational methods of geometric mechanics to the phenomenological description of educational and psychological assignments of optimal control finding for the Hamiltonian dynamic system of a student trying to learn new educational material [26]. Lewis draws attention to the important practical circumstance that computational problems of optimal control with instantaneous jump-like switching of

the step profile of the control signal “at stops” as a rule abstractly model extremely undesirable shock-maximalist approaches to teaching schoolchildren and students. These bang-bang-shaped control approaches contradict all existing psychological, pedagogical as well as medical and biological recommendations regarding the need to ensure the smoothness of switching of the control signal in all processes of knowledge acquisition and instruction [26]. Taking into account the above remark formulated in the article [26], it can be further suggested here that the high-intensity bang-bang approaches to STEM learning with instantaneous abrupt switching of control signals mentioned in [26] can probably be practically used primarily for the purposes of immediate improvement of engineering and technical skills as part of the continuous professional development of working professionals who are generally more resistant to oscillations of the control signal. Such time parameters as free time for unlimited independent study of a new course by a student or fixed semester time for planned synchronous or asynchronous study of a new academic discipline, strictly regulated by curricula and a report card-calendar, are not mentioned at all and are not considered either explicitly or implicitly in either formulas (1)–(39) or in Figs. 1–4 in the optimisation work [26]. It is due to the entirely practical reason of the systematic ignoring by most modern students of any formal deadlines for the timely study of the minimum theoretical foundations of a new course and due to the actual failure of a participant in the educational process to complete the minimum sufficient volume of individual assignments sufficient to obtain a minimum positive grade in a new discipline [26].

In the work [27], analytical and computational methods of optimal control theory were used within the framework of multi-criteria optimisation analysis of a nonlinearly controlled four-block deterministic dynamic system consisting of university students studying mathematics, psychologically infected with their hostile attitude towards the need for timely study of mathematical university disciplines with the presence of two independent signals of organisational and psychological control  $\{u_1, u_2\}$  and a nonlinear objective function in the form of a minimized integral quadratic quality criterion  $J(u_1, u_2)$ , where the first control signal  $u_1$  characterises organisational efforts to prevent the spread of a hostile attitude towards the study of mathematics, and the second control signal  $u_2$  characterises organisational efforts to reduce the duration of time of individual psychological recovery of a student from an acquired hostile attitude towards the study of mathematics [27]. As part of the implementation of the optimisation approach by Teklu & Terefe [27], the time dependencies of the phase variables characterising the epidemic states of psychological “susceptibility” to mathematical hostility, social “exposure” to hostility towards the studied mathematics course, psychological “infection” with hostility towards mathematics and the subsequent heutigologic “treatment” of modern university students from an unfriendly attitude towards the studied mathematical disciplines were computationally modelled and graphically visualised. This allowed the authors [27] to successfully carry out an optimisation educational and

psychological study of nonlinear socio-dynamic effects associated with the implementation of organisational and psychological efforts to reduce the hostile attitude of university students to the need for the regular study of mathematical disciplines during 6.67 years (80 months) of university study of mathematics (in Fig. 6), during 12.5 years (150 months) of university mastering of mathematics (in Figs. 2–5, 8), during 16.67 years (200 months) of studying mathematics at the university (in Fig. 9), and also during 20.83 years (250 months) of university mastering of mathematics (in Fig. 7). It should be noted that the long-term periods of successful overcoming of hostile attitude towards mathematical disciplines by university students during many years of studying a university course of mathematics in the range from 6.67 years (Fig. 6) to 20.83 years (Fig. 7), analysed in [27], significantly exceed both the four-year terms of university education for school graduates at the Bachelor's degree program and the two-year terms of university education for bachelors at the Master's degree program. Thus, it can be concluded that the results of the authors' computational modelling either correspond to certain individual trajectories of twenty years of university education with in-depth study of one or two mathematical disciplines in each semester, or still describe a nonlinear social dynamics of a two-decade psychological overcoming of individual hostile attitude towards mathematical disciplines within the framework of continuous professional development in lifelong engineering education, which obviously emphasises the actual discrepancy between the publication topic and the authors' research results in the computational work [27].

El Bhih et al. [30] used optimisation-epidemic methods of applied mathematics within the framework of the system-dynamic statement (Fig. 1) and subsequent numerical solution (Figs. 2, 3, 5–7) of the psychological and pedagogical problem of optimal control of different group levels of student knowledge in the mathematics course. The authors modelled five groups of university students: university freshmen; weak students who are constantly lagging behind in mathematics and study mainly with grades {E–D} in mathematics; stronger students—good students with grades {C–B} in mathematics; excellent students with grades {A} in mathematics, who are able to successfully solve applied problems; and also excellent students with grades {A} in mathematics who experience difficulties in solving applied problems (Figs. 1–7). The graphs of all transition processes for the level of knowledge of mathematically weak and lagging students (Figs. 2, 3, 5–7) reach their stationary values starting from the minimum three-week period of continuous study of mathematics during the first 20 days (Fig. 7) of in-depth study or, on average, during a seven-week continuous period of study of mathematics during the first 50 days (Figs. 2, 3), which additionally confirms the practical unrealisticness and speculative nature of any “quick strategies” for high-speed and high-quality study of a university course in mathematics “at the last moment”. It turns out that to receive a “solid” satisfactory grade, a university course in mathematics still needs to be studied from 3 to 7 weeks [30].

The paper by Åkesson et al. [36] was cited as the first reference in the optimisation textbook [24], [25] because Teo et

al. (2021) factually encouraged their students to widely use computational possibilities of free optimisation software JModelica.org with Optimica extension for numerical solution of optimal control problems. Hydraulic and electrical analogies for a simple description of student learning dynamics were also proposed in the educational papers [37]–[41].

In the scientific work [42], the presence of more than 400 pedagogically satisfactory, psychologically acceptable and practically effective educational strategies of student learning was established. The strategies were applied, to one degree or another, by education seekers for the full or partial achievement of the set educational goals and objectives of any successful and, if possible, systematic learning [42]. The publication by Hattie & Donoghue especially emphasises the importance of further searching, first of all, for optimal educational strategies [42], which further stresses the relevance of the presented author's study devoted to the rational construction of nonlinear target functions describing various strategies of dynamic behaviour of a student studying a new course.

## II. AIM, RESEARCH TASKS AND PRIME SCIENTIFIC NOVELTY

Relevance of the research: The choice of an optimal educational strategy that ensures minimal sufficient efficiency in the study of a new course by a student is one of the “eternal” didactic tasks of both pedagogy and educational psychology, as well as machine learning and robotics.

It is possible to strengthen the course of engineering education with {Modelica-&-Optimica}-enhanced computational research into the  $J$ -performance index-dependent learning dynamics of a lazy and forgetful student taking into account that the  $J$ -objective function-dependent educational activity was not comprehensively addressed in previous research studies.

The socio-technical task of finding a non-linear optimal educational approach for the methodologically effective study of a new course is relevant for the successful planning of both the university education of a student and the continuous professional development of a working engineer.

The object of the study is a non-linear educational and psychological process of individual study of a new academic course by a student during a semester, implemented under the assumption that the student actually achieves a certain final level of knowledge, sufficient to receive a positive assessment within the framework of successful study of a new academic discipline.

The subject of the research is the educational and psychological characteristics of non-stationary transition processes that arise during the controlled study of a new course by a student, depending on the engineering-designed algebraic structure of the minimized functional that determines the individual educational trajectory of the student's study of a new discipline within the framework of a certain trend of student behaviour.

The aim of the research is the author's attempt to establish pedagogically acceptable educational and psychological patterns within the framework of optimisation nonlinear-

dynamic modelling of the educational behaviour of a student studying a new course, based on a non-obvious generalization of various educational trajectories depending on the algebraic structure of nonlinear target functions constructed by the author in the form of various minimized functionals.

The novelty of the research: In the context of solving boundary value problems on the dynamics of optimal control of the educational and psychological process of studying a new course by a student, the engineering understanding of the nature of the influence of the algebraic structure of the minimized functional on the nonlinear dynamic features of transient processes characterising the individual educational strategy for studying a new academic discipline by a student has been expanded.

Optimica-simulation description of pedagogically possible educational approaches to studying a new course by a student and a working engineer with varying degrees of personal interest and actual involvement in a specific heutagogic process of direct study of the current new course was carried out within the framework of engineering Modelica-design of sufficiently complex nonlinear objective functions for cybernetic modelling of student educational behaviour to ensure the possibility of computational accounting and accessible graphical visualisation of nonlinear effects of the influence of different goal-setting of the subject of study on the phenomenological dynamics of the student's study of a new course during a predetermined time frame, but with implicit consideration of a certain student background, which differs significantly for students who are currently at different emotional and psychological, professional or biological age stages of their continuous engineering education.

### III. RESEARCH METHODOLOGY

The optimisation Modelica modelling of student educational strategies No. 1–17, as shown in Figs. 1–17, was performed within the framework of using the open source software JModelica.org with the Optimica extension [36]. The Modelica simulation results in the form of an archived directory with the corresponding \*.mop, \*.py and \*.png files were uploaded as a dataset to the ResearchGate platform [44].

### IV. DYNAMIC EQUATION FOR STUDENT LEARNING DYNAMICS DESCRIPTION

Effectively studying a new course within a pre-planned time frame is an important practical task for both the student studying a new subject as part of a university degree and the working professional studying a new course as part of a formal certificate of successful completion of the course. On the other hand, if we compare the available socio-economic resources of a student and a working engineer, we can note that a student has significantly more free time to study a new discipline than an engineer, and a working professional has a significantly higher level of motivation to study a new course than the average student. As a result, both the goals and approaches to studying a new course by a student and an engineer will differ significantly.

Let us try to formulate a metaphorical analogy that would give a schematic hydrodynamic representation of the very idea of an optimisation model of student learning dynamics (Figs. 1–17). Let a student who is currently trying to study a certain new course be an imaginary “tub” partially filled with “water” of new knowledge, and the current level of “liquid” of new knowledge in this “tub” changes in a certain way during the student's study of the new course. Let an “external” or “internal” teacher be an imaginary “aquarius” who from time to time “adds” a certain amount of “liquid” of new knowledge to the “barrel” of knowledge from the new course, actually ensuring the implementation of some educational (or self-educational) strategy of “external” or “internal” control of the level of filling the “barrel” of the student's memory with “liquid” of knowledge from the new course, and the control of the learning process by the teacher is the only control signal in our problem.

Let the lifelong partial forgetting of a part of the newly learned new educational material by a student be a kind of “thief” or a Stephen King's “langollier” who is always thirsty for “water” of new knowledge in the “barrel” of knowledge, who is constantly turning on the partially closed “tap” of forgetting at the bottom of the “barrel” of new knowledge. The higher the available level of “liquid” of new current knowledge in the “barrel” of student memory, the more intense the forgetting of the newly learned material of the new course. It is due to the greater outflow-discharge of the “liquid” of knowledge through the more open “tap” of forgetting of the freshly learned content, and taking into account the effect of forgetting does not imply the use of the second control signal in the model.

We will assume that the choice of a particular optimisation strategy of a specific methodological approach to organising the effective way of student learning (Figs. 1–17) occurs not at the level of the student's memory “barrel”, partially filled with the “liquid” of new knowledge, but at some higher level of making rational management decisions by an external or internal student teacher – “aquarius”, who chooses one or another “hydrodynamic” approach to continuously filling the “barrel” of student memory with the “liquid” of the content of the new educational course.

The simple optimisation model of forgetful student learning dynamics was briefly mentioned as Example 1.2.1 on pp. 2 and 3 of Chapter 1 [24] and as Example 6.4.2 on pp. 186 and 187 of Chapter 6 [25] in the international textbook [24], [25]:

$$\left. \begin{array}{l} \left( \min_{u(t)}(J) \right) \text{ subject to the dynamics:} \\ \left( \frac{d}{dt}(x_1(t)) \right) = b \cdot u(t) - c \cdot x_1(t); \\ \left( (x_1(0)) = x_{10}; (x_1(t_f)) = x_{1f} \right); \\ \left( 0 \leq u(t) \leq u_{\max}; (0 \leq t \leq (t_f)) \right); \end{array} \right\} \quad (1)$$

where

- the objective function ( $J$ ) is the minimized functional of a previously unknown algebraic structure that determines the individual student approach to instructor-guided dynamics of

the course learning process, where mathematical  $J$ -structure must be comprehensively specified for the successful numerical solution of the particular optimal control problem (1);

- the first phase (state) variable  $x_1(t)$  is the current level (volume) of student-acquired knowledge of a new course studied by a student;

- the state – the initial value  $x_{10}$  of the first phase variable  $x_1(0)$  is the numerical value of the state variable  $x_1(t_0)$  for the start time moment  $t_0 = 0$ , i.e.,  $x_1(0)$  is the initial level of the studied course knowledge;

- $(t_f) = T$  is the duration of the course learning time, e.g., the duration of the university semester;

- the constraint – the final value  $x_{1f}$  of the first phase variable  $x_1(t_f)$  is the numerical value of the state variable  $x_1(T)$  for the final time moment  $(t_f) = T$ , i.e.,  $x_1(T)$  is the final level of the studied course knowledge, where  $x_1(T) > x_1(0)$  for successful course learning and the minimum academically acceptable final value of a satisfactory course grade must be higher than 55 (55/100) ECTS-grades “E”, i.e.,  $x_1(T) \geq 0.55$ ;

- the additional overdot parameter  $x_2(t) = (d(x_1(t))/dt) = (x_1(t))' = \dot{x}_1(t)$  is the time rate of change of student-acquired knowledge level (volume)  $x_1(t)$  of a new course studied by a student;

- from the technical instructor’s viewpoint, the bounded control signal  $u(t)$  determines the instructor-driven rate of student knowledge acquisition or, in other words, the instructor-guided intensity of student knowledge inflow into student memory, where  $0 \leq u(t) \leq u_{\max}$  and  $u_{\max}$  are the maximum value of control signal;

- from the student’s viewpoint, the restricted control signal  $u(t)$  determines the student-chosen rate of student knowledge acquisition or, in other words, the student-guided intensity of student knowledge inflow into student memory, where  $0 \leq u(t) \leq u_{\max}$  and  $u_{\max}$  are the upper bound for the control signal;

- the coefficient  $b$  is the efficiency coefficient of new knowledge acquisition or, in other words, the utilisation factor of inflowing knowledge understanding;

- the first positive term  $(b \cdot u(t))$  is the time rate of new knowledge acquisition by a student or, in other words, the intensity of student-acquired new knowledge inflow into student memory;

- the second negative term  $-(c \cdot x_1(t))$  is the time rate of new knowledge forgetting or, in other words, the intensity of student-acquired new knowledge outflow from student memory;

- the student knowledge balance equation  $(d(x_1(t))/dt) = b \cdot u(t) - c \cdot x_1(t)$  means that the time rate of change of the current volume (level) of the student knowledge is dynamically determined by  $b$ -efficient and  $u(t)$ -driven the inflow flow rate of student-acquired course knowledge into student memory and  $c$ -defined the outflow flow rate of student-forgotten course knowledge from student memory, where student learning efficiency factor  $b$  must be larger than knowledge forgetting factor  $c$ , i.e.,  $b > c$  for academically successful course learning.

Let us assume the following numerical values for the computer modelling of optimal control problem (1) with JModelica.org- {1.17; 2.14} optimisation software:

$$\left. \begin{aligned} b = 0.5; c = 0.2; (t_f) = T = 15; \\ (0 \leq u(t) \leq 12); (0 \leq t \leq T); \\ ((x_1(0)) = 0.1); ((x_1(T)) = 0.9); \end{aligned} \right\} \quad (2)$$

where we assume that the student successfully acquired knowledge required by the course from the “zero” – starting level of 10 grades ( $x_{10} = (x_1)_{\text{start}} = 10/100$ ) to the excellent “hero” –final level of 90 grades ( $x_{1T} = (x_1)_{\text{final}} = 90/100$ ).

It is a nonlinear computational assignment to numerically solve student learning optimisation problem (1)–(2) for the different {Modelica, Optimica}-solvable  $J$ -structures (see Table I on the next page) and additionally classify Modelica-derived numerical solutions into the particular student-chosen educational strategies of new course learning.

TABLE I  
NONLINEAR OBJECTIVE FUNCTIONS FOR THE DESCRIPTION OF STUDENT LEARNING STRATEGIES

A) Student Course-Learning Learning Strategy A “Lazy Student”: Late Final Learning Activity (Figs. 1 and 2)	
A1)	$\min_{u(t)}(J_1) = \min_{u(t)} \left( \int_0^T (u(t)) dt \right), (3)$ , where $T$ is term duration in weeks (Fig. 1)
A2)	$\min_{u(t)}(J_2) = \min_{u(t)} \left( \int_0^T \left( (x_1(t))^2 - \left( \frac{1}{2} \cdot (u(t))^2 \right) \right) dt \right), (4)$ , where $T$ is term duration in weeks (Fig. 2)
B) Student Course-Learning Learning Strategy B “Procrastinator”: Delayed Learning Activity (Figs. 3 and 4)	
B1)	$\min_{u(t)}(J_3) = \min_{u(t)} \left( \int_0^T (u(t)) \cdot \left( \exp((x_1(t))^2 + (x_2(t))^2 + (u(t))^2) \right) dt \right), (5)$ , where $T$ is term duration in weeks (Fig. 3)

B2) $\min_{u(t)}(J_4) = \min_{u(t)} \left( \int_0^T (u(t) + (u(t))^2 + (u(t))^3 + (u(t))^4) dt \right)$ , (6), where $T$ is term duration in weeks (Fig. 4)
C) Student Course-Learning Strategy C “Growing Student”: Monotonic Growing Learning Activity (Figs. 5 and 6)
C1) $\min_{u(t)}(J_5) = \min_{u(t)} \left( \int_0^T ((x_1(t))^2) \cdot ((x_2(t))^2) \cdot (u(t))^2 dt \right)$ , (7), where $T$ is term duration in weeks (Fig. 5)
C2) $\min_{u(t)}(J_6) = \min_{u(t)} \left( \int_0^T \left( (x_1(t))^3 - u(t) \right)^2 \cdot \left( \frac{d(x_1(t))}{dt} \right)^6 dt \right)$ , (8), where $T$ is term duration in weeks (Fig. 6)
D) Student Course-Learning Strategy D “Steady Student”: Ongoing Learning Efforts Activity (Fig. 7)
D1) $\min_{u(t)}(J_7) = \min_{u(t)} \left( \int_0^T ((-1) \cdot (x_1(t) - u(t)))^4 \cdot (\exp((-1) \cdot x_1(t))) dt \right)$ , (9), where $T$ is term duration in weeks (Fig. 7)
E) Student Course-Learning Strategy E “Midterm Hero” or “Halfway Hero”: Maximum Midterm Learning Activity (Figs. 8–11)
E1) $\min_{u(t)}(J_8) = \min_{u(t)} \left( \int_0^T (u(t) - 1)^2 \cdot (\exp((-1) \cdot x_1(t))) dt \right)$ , (10), where $T$ is term duration in weeks (Fig. 8)
E2) $\min_{u(t)}(J_9) = \min_{u(t)} \left( \int_0^T ((-1) \cdot (x_1(t) - u(t)))^3 \cdot (\exp((-1) \cdot x_1(t))) dt \right)$ , (11), where $T$ is term duration in weeks (Fig. 9)
E3) $\min_{u(t)}(J_{10}) = \min_{u(t)} \left( \int_0^T ((-1) \cdot (x_1(t) - u(t)))^4 \cdot (\exp((-1) \cdot x_1(t))) dt \right)$ , (12), where $T$ is term duration in weeks (Fig. 10)
E4) $\min_{u(t)}(J_{11}) = \min_{u(t)} \left( \int_0^T \left( (-1) \cdot \left( \frac{1}{x_1(t)} \right) \cdot (u(t))^2 \right) dt \right)$ , (13), where $T$ is term duration in weeks (Fig. 11)
F) Student Course-Learning Strategy F “Starting Hero” or “Sprinting Hero”: Intensive Start Learning Activity (Figs. 12–17)
F1) $\min_{u(t)}(J_{12}) = \min_{u(t)} \left( \int_0^T (u(t) - x_1(t)) dt \right)$ , (14), where $T$ is term duration in weeks (Fig. 12)
F2) $\min_{u(t)}(J_{13}) = \min_{u(t)} \left( \int_0^T ((u(t))^2) - ((x_1(t))^2) dt \right)$ , (15), where $T$ is term duration in weeks (Fig. 13)
F3) $\min_{u(t)}(J_{14}) = \min_{u(t)} \left( \int_0^T ((u(t))^3) - ((x_1(t))^3) - ((x_2(t))^3) dt \right)$ , (16), where $T$ is term duration in weeks (Fig. 14)
F4) $\min_{u(t)}(J_{15}) = \min_{u(t)} \left( \int_0^T \left( (x_1(t))^3 - u(t) \right) \cdot \left( (x_2(t))^3 - u(t) \right) \cdot \left( \frac{d(x_1(t))}{dt} \right)^6 dt \right)$ , (17), where $T$ is term duration in weeks (Fig. 15)
F5) $\min_{u(t)}(J_{16}) = \min_{u(t)} \left( \int_0^T \left( \left( -\frac{1}{2} \right) \cdot ((x_1(t))^2 + (x_2(t))^2 + (u(t))^2) \right) dt \right)$ , (18), where $T$ is term duration in weeks (Fig. 16)
F6) $\min_{u(t)}(J_{17}) = \min_{u(t)} \left( \int_0^T \left( \left( -\frac{1}{2} \right) \cdot ((u(t))^2 + 1) \right) dt \right)$ , (19), where $T$ is term duration in weeks (Fig. 17)

V. STUDENT COURSE-LEARNING STRATEGY A “LAZY STUDENT”: LATE FINAL LEARNING ACTIVITY

Student late learning strategy “A” is outlined in Figs. 1 and 2.

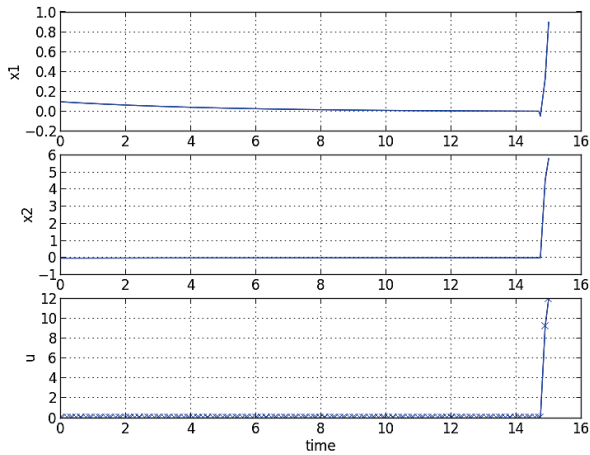


Fig. 1. Educational strategy No. 1 of the study behaviour of a lazy student who does nothing every time during the entire semester and begins to work intensively only in the last few days immediately before the session.

Fig. 1 shows the first course-learning strategy with the late final learning activity of an academically lazy student which is determined by the optimal control problem (1) and (2) with the minimized objective function (3), where the first minimized integral functional (A1) is defined as a minimum of a definite integral of the control signal over the time interval during which the student studies the new course, i.e.,  $\min(J_1) = \min(\int(0 \text{ to } T) (u(t)) dt)$ .

Fig. 1 corresponds to the integral performance index (3), (A1) and shows that lazy student does nothing until the final exam disregarding the course duration, e.g., student educational inactivity took place at least in the first 14.5 weeks of a 15-week-long course. The lazy student shows the strong intention to “study” all course material only in the last 2–3 days before examination. It is obvious that the lazy student’s “educational” behaviour would show complete inactivity in the first 49.5 weeks of a 50-week-long course, etc.

Fig. 2 is the second course-learning strategy with the late final learning activity of an academically lazy student which is determined by the optimal control problem (1) and (2) with the minimized objective function (4), where the second minimized integral functional (A2) is defined as a minimum of a definite integral of the difference of the square of the first phase variable minus half the square of the control signal over the time interval during which the student studies the new course, i.e.,  $\min(J_2) = \min(\int(0 \text{ to } T) (((x_1(t))^2) - (1/2) \times ((u(t))^2))dt)$ .

Fig. 2 corresponds to the integral performance index (4), (A2) and shows that the lazy student can express some initial activity during the first week of the course and then completely drops from educational process for the remaining 14 weeks until the final examination. As usual all “learning” activity took place in the last 2–3 days of a 15-week-long course.

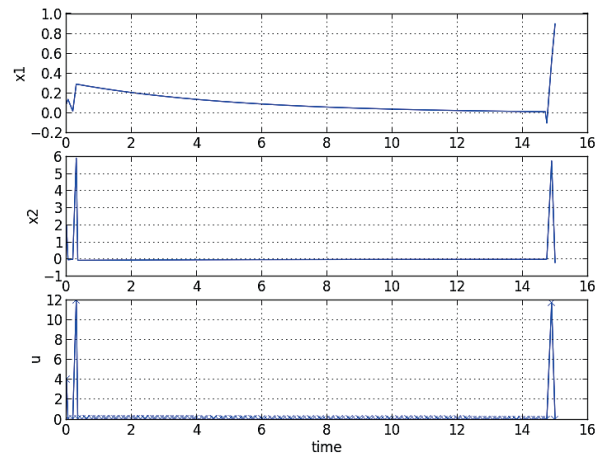


Fig. 2. Educational strategy No. 2 of the learning behaviour of a lazy student who, at the beginning of studying a new course, demonstrates some initial interest in the subject of classes and the teaching style of the new instructor, and then, out of the old habit of doing nothing, drops out of the current learning process for a long time until the very end of the semester with the usual pre-exam activation of pre-session learning activity in the last few days of the course.

It should be noted that bad organisational and behavioural habits No. 1–No. 2 of escaping from fulfilling educational or work responsibilities, associated with an attempt to “successfully” complete the entire necessary volume of planned work exclusively in the last few days of an educational or production project can characterise the late executive activity of not only a lazy student, but also a lazy Web programmer, lazy engineer, lazy teacher, lazy researcher, etc., due to the fact that strategies No. 1–No. 2 of organisational behaviour to a greater extent characterise the psychological, if not mental, fatigue of the subject from the current work routine and an attempt to save energy for alternative non-work activities, for example, for many years of simultaneous work for several employers or for continuous collecting of diplomas of higher education or for round-the-clock computer games, etc. They often turn out to be an even more boring routine, but consciously chosen alternatives, nevertheless, seem to be something more significant or deserving more attention to subjects who consciously prefer behavioural strategies No. 1 and No. 2.

VI. STUDENT COURSE-LEARNING STRATEGY B “PROCRASTINATOR”: DELAYED LEARNING ACTIVITY

Student delayed learning strategy “B” is outlined in Figs. 3 and 4.

Fig. 3 shows the third course-learning strategy with delayed learning activity of an academically lazy student which is determined by the optimal control problem (1)–(2) with the minimized objective function (5), where the third minimized integral functional (B1) is defined as the minimum of a definite integral of the product of the control signal multiplied by the number  $e$  raised to a power, where the exponent is defined as (the sum of the square of the first phase variable plus the square of the second auxiliary variable plus the square of the control signal) over the semester time, i.e.,  $\min(J_3) = \min(\int(0 \text{ to } T) ((u(t)) \times (\exp(((x_1(t))^2) + ((x_2(t))^2) + ((u(t))^2))))dt)$ .

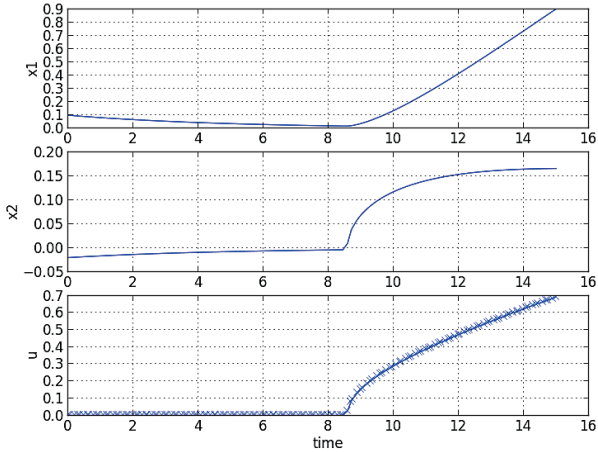


Fig. 3. Educational strategy No. 3 of the learning behaviour of a constantly busy procrastinating student who does nothing for the first 10 weeks of the semester and then begins to study during the last 5 weeks of the course.

Fig. 3 corresponds to the integral performance index (5), (B1) and shows that a busy student does nothing in the first 10 weeks of a 15-week-long course. The busy student shows a strong intention to study all course material only in the last 5 weeks before the examination.

Fig. 4 shows the fourth course-learning strategy with delayed learning activity of an academically lazy student which is determined by the optimal control problem (1)–(2) with the minimized objective function (6), where the fourth minimized integral functional (B2) is defined as a minimum of a definite integral of the sum of the control signal with the addition of the square of the control signal with the addition of the fourth power of the control signal over the time interval during which the student studies the new course, i.e.,  $\min(J_4) = \min(\int(0 \text{ to } T) ((u(t)) + ((u(t))^2) + ((u(t))^3) + ((u(t))^4)dt)$ .

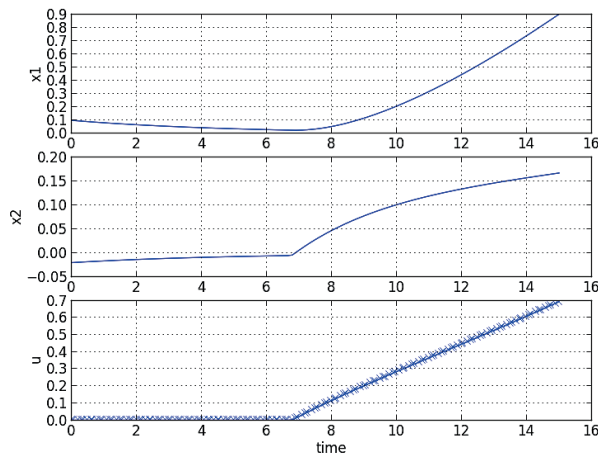


Fig. 4. Educational strategy No. 4 of the learning behaviour of a constantly busy procrastinator student who does nothing for the first 9 weeks of the semester and then begins to study during the last 6 weeks of the course.

Fig. 4 corresponds to the integral performance index (6), (B2) and shows that a busy student does nothing in the first 9 weeks of a 15-week-long course. The busy student shows the strong intention to study all course material only in the last 6 weeks before the examination.

It is obvious that “B”-case (Figs. 3 and 4) are better than “A”-case (Figs. 1 and 2) because it is better to study the course for five weeks than 3 days. However, the course duration is 15 weeks and more computational endeavours are needed to identify more sustainable course learning trends.

It should be noted that there is an increasing popularity among schoolchildren, as well as students of both junior and senior courses, of both procrastination strategies No. 3–No. 4 of delayed completion of academic assignments with the start of any academic activity mainly in the last third of the course, and “lazy” strategies No. 1–No. 2 of late completion of necessary work with the start of the “study” process exclusively “at the last moment” before the exam. The growing popularity among the youth audience of quasi-“educational” procrastination strategies No. 1–No. 4 of quickly “studying” the next new course may be associated with the sincere conviction of the majority of overloaded schoolchildren, students, teachers, methodologists, programmers and researchers that modern high-tech “study” should not take up too much personal time, and their direct educational responsibilities for regular and systematic study, as well as individual elaboration of the educational materials of the new course. The routine process of completing the “boring” and “useless” calculation-graphic and coursework assignments on the new discipline can be delegated everywhere and on a permanent basis to numerous systems of generative artificial intelligence, often even without additional verification of the correctness of the content generated by chatbots, with a sincere, but somewhat naive, belief that the simultaneous use of high-speed simulation-computing capabilities of the ChatGPT, Claude, Copilot, DeepSeek, Gemini, Grok, Perplexity, etc., are capable of successfully solving all current learning and instructional problems of the educational community within the framework of detached-indifferent educational and behavioural strategies No. 1–No. 4.

VII. STUDENT COURSE-LEARNING STRATEGY C “GROWING STUDENT”: MONOTONIC GROWING LEARNING ACTIVITY

Student monotonic growing learning strategy “C” is outlined in Figs. 5 and 6.

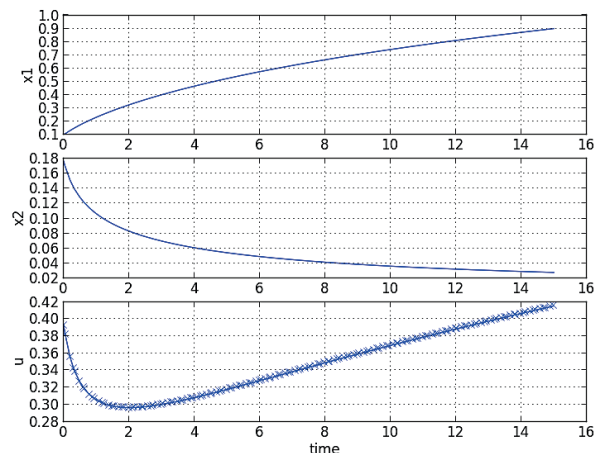


Fig. 5. Educational strategy No. 5 of the learning behaviour of an academically diligent and hardworking student, whose level of knowledge monotonically increases throughout all 15 weeks of studying a new course without oscillations of the control signal.

Fig. 5 shows the fifth course-learning strategy with monotonic growing learning activity of an academically lazy student which is determined by the optimal control problem (1)–(2) with the minimized objective function (7), where the fifth minimized integral functional (C1) is defined as a minimum of a definite integral of the product of the square of the first phase variable, multiplied by the square of the second auxiliary variable, multiplied by the square of the control signal over the semester time, i.e.,  $\min(J_5) = \min(\int(0 \text{ to } T) ((x_1(t))^2 \times (x_2(t))^2 \times (u(t))^2) dt)$ .

Fig. 5 shows that the student acquires the first half of the new course in the first 4.5 weeks, and the second half of the new course is studied in the next 10.5 weeks of a 15-week-long semester. Fig. 5 corresponds to the integral performance index (7), (C1), and Fig. 6 corresponds to the integral performance index (8), (C2). Figs. 5 and 6 show a more sustainable learning trend “C” when a course-enrolled student is working during all 15 weeks of the course duration. However, student learning is rather uneven.

Fig. 6 shows that the student acquires the first half of a new course in the first 6 weeks, and the second half of a new course will be studied in the next 9 weeks of the semester.

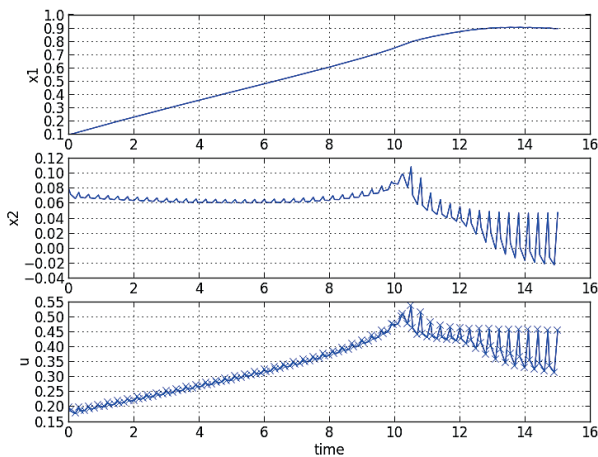


Fig. 6. Educational strategy No. 6 of the learning behaviour of an academically diligent and hardworking student, whose level of knowledge monotonically increases throughout all 15 weeks of studying a new course with additional oscillations of the control signal.

Fig. 6 shows the sixth course-learning strategy with monotonic growing learning activity of an academically lazy student which is determined by the optimal control problem (1)–(2) with the minimized objective function (8), where the sixth minimized integral functional (C2) is defined as a minimum of a definite integral of the product of the sixth power of the first time derivative of the first phase variable, multiplied by the square of the difference of the cube of the first phase variable minus the control signal over the time interval during which the student studies the new course, i.e.,  $\min(J_6) = \min(\int(0 \text{ to } T) (((x_1(t))^3 - u(t))^2 \times (d/dt(x_1(t)))^6) dt)$ .

However, the student course learning rate may vary with different individuals and external factors. Therefore, additional computational endeavours with the student performance index are important.

### VIII. STUDENT COURSE-LEARNING STRATEGY D “STEADY STUDENT”: ONGOING LEARNING EFFORTS ACTIVITY

Student ongoing learning efforts strategy “D” is shown in Fig. 7. Fig. 7 corresponds to the integral performance index (9), (D1).

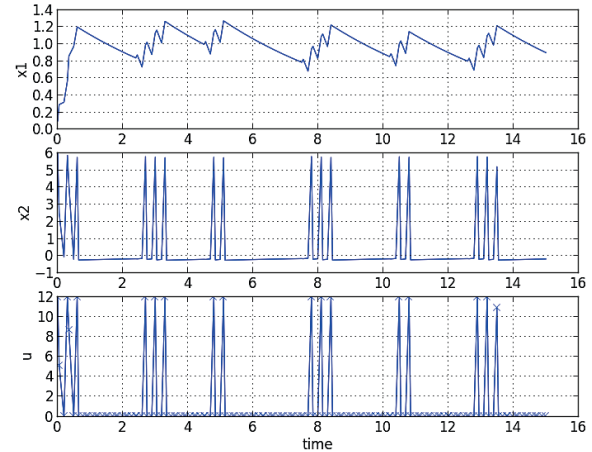


Fig. 7. Educational strategy No. 7 of the learning behaviour of an academically diligent and hardworking student, whose sufficient level of knowledge is constantly maintained at a sufficiently high level throughout the entire academic semester.

Fig. 7 shows the seventh course-learning strategy with the ongoing learning efforts activity of a regularly-working student which is determined by the optimal control problem (1) – (2) with the minimized objective function (9), where the seventh minimized integral functional (D1) is defined as the minimum of a definite integral of the product of the difference of the control signal minus the first phase variable, multiplied by the number  $e$  raised to a power, where the exponent is defined as (the first phase variable taken with a negative sign) over the time interval during which the student studies the new course, i.e.  $\min(J_7) = \min(\int(0 \text{ to } T) ((-1) \times (x_1(t) - u(t))^{1 \times (\exp((-1) \times x_1(t)))) dt)$ .

The regular learning activity of a hard-working student, in this case, is a sloping sawtooth curve of small oscillations of the student’s current level of knowledge relative to the desired final level of knowledge, which continue throughout the entire course of study (Fig. 7).

### IX. STUDENT COURSE-LEARNING STRATEGY E “MIDTERM HERO” OR “HALFWAY HERO”: MAXIMUM MIDTERM LEARNING ACTIVITY

Student maximum midterm learning strategy “E” is outlined in Figs. 8–11. Fig. 8 shows the eighth course-learning strategy with the maximum midterm learning activity of a hard-working student which is determined by the optimal control problem (1)–(2) with the minimized objective function (10), where the eighth minimized integral functional (E1) is defined as the minimum of a definite integral of the product of the square of the difference of the control signal minus one, multiplied by the number  $e$  raised to a power, where the exponent is defined as (the first phase variable taken with a negative sign) over the time interval during which the student studies the new course,

i.e.,  

$$\min(J_8) = \min(\int(0 \text{ to } T) ((u(t) - 1)^2 \times (\exp((-1) \times (x_1(t))))))dt).$$

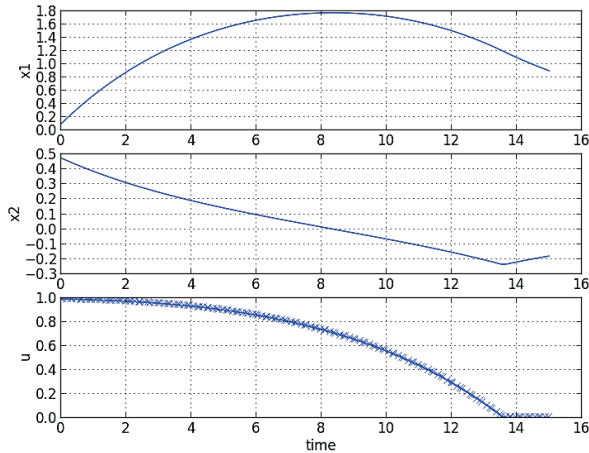


Fig. 8. Educational strategy No. 8 of the learning behaviour of an academically diligent and hardworking student who studies quite intensively only during the first 8 weeks of the semester and then rests for the next 7 weeks of the academic semester, which remain until the examination session.

Fig. 9 shows the ninth course-learning strategy with the maximum midterm learning activity of a hard-working student which is determined by the optimal control problem (1)–(2) with the minimized objective function (11), where the ninth minimized integral functional (E2) is defined as the minimum of a definite integral of the product of the cube of the difference of the first phase variable minus the control signal, multiplied by minus one, multiplied by the number  $e$  raised to a power, where the exponent is defined as (the first phase variable taken with a negative sign) over the semester-long time interval during which the course-enrolled student acquires the new discipline, i.e., 
$$\min(J_9) = \min(\int(0 \text{ to } T) ((-1) \times (x_1(t) - u(t))^3 \times (\exp((-1) \times (x_1(t))))))dt).$$

Fig. 8 corresponds to the integral performance index (9), (E1). Fig. 9 corresponds to the integral performance index (10), (E2).

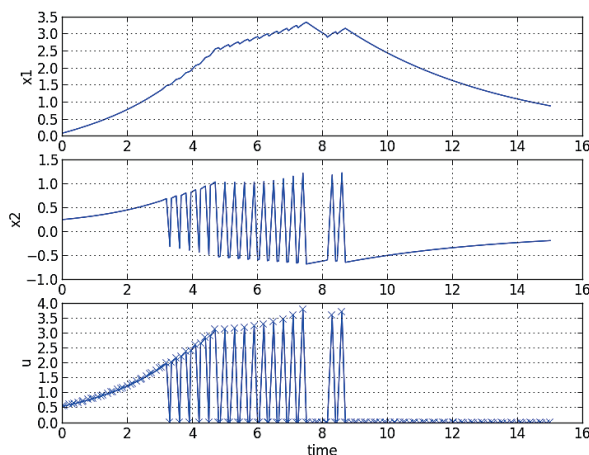


Fig. 9. Educational strategy No. 9 of the learning behaviour of an academically diligent and hardworking student who studies quite intensively only during the first 7.5 weeks of the semester and then rests for the next 7.5 weeks of the academic semester, which remain until the examination session.

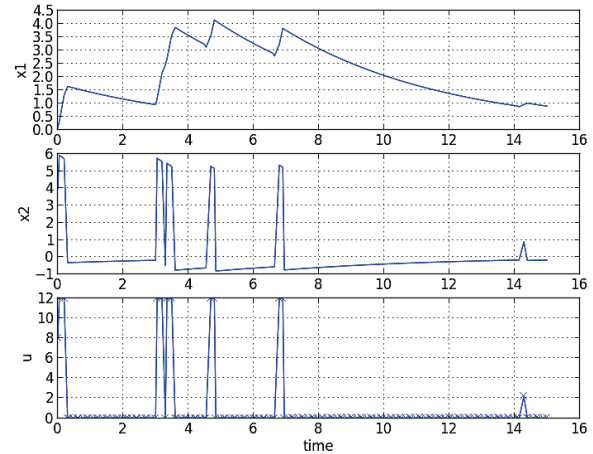


Fig. 10. Educational strategy No. 10 of the learning behaviour of an academically diligent and hardworking student who studies quite intensively only during the first 5 weeks of the semester and then rests for the next 10 weeks of the academic semester, which remain until the examination session.

Fig. 10 shows the tenth course-learning strategy with the maximum midterm learning activity of a hard-working student which is determined by the optimal control problem (1)–(2) with the minimized objective function (12), where the tenth minimized integral functional (E3) is defined as the minimum of a definite integral of the product of the fourth degree of the difference of the first phase variable minus the control signal, multiplied by minus one, multiplied by the number  $e$  raised to a power, where the exponent is defined as (the first phase variable taken with a negative sign) over the semester-long time interval during which the course-enrolled student acquires the new discipline, i.e., 
$$\min(J_{10}) = \min(\int(0 \text{ to } T) ((-1) \times (x_1(t) - u(t))^4 \times (\exp((-1) \times (x_1(t))))))dt).$$

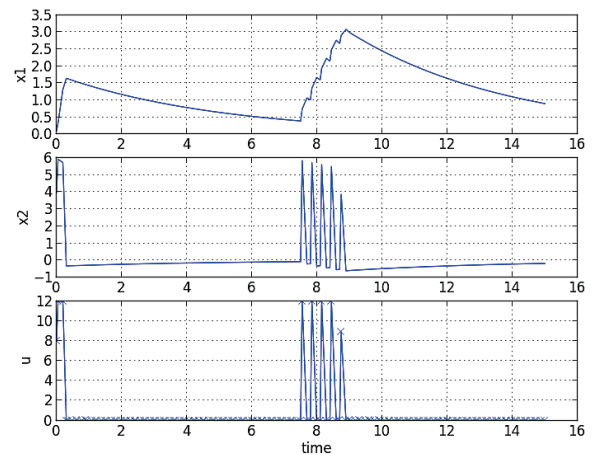


Fig. 11. Educational strategy No. 11 of the learning behaviour of an academically diligent and hardworking student who studies quite intensively only during the first 9 weeks of the semester and then rests for the next 6 weeks of the academic semester, which remain until the examination session.

Fig. 10 corresponds to the integral performance index (11), (E3). Fig. 11 corresponds to the integral performance index (12), (E4).

Fig. 11 shows the eleventh course-learning strategy with the maximum midterm learning activity of a hard-working student which is determined by the optimal control problem (1)–(2) with the minimized objective function (13), where the eleventh

minimized integral functional (E4) is defined as the minimum of a definite integral of the product of the square of the control signal, multiplied by the inverse of the first phase variable, multiplied by minus one over the time interval during which the student studies the new course, i.e.,  $\min(J_{11}) = \min(\int(0 \text{ to } T) ((-1) \times (1/(x_1(t)))) \times ((u(t))^2) dt)$ .

Fig. 8 shows that a hard-working student intensively studies both mandatory and facultative course topics but only in the first 8 weeks of a 15-week-long course. In the first 8 weeks the course-devoted student manages successfully to complete two times more course-proposed assignments than it is required for the standard “excellent” course grade. However, all further course-related insights disappear in the last 7 weeks of further student’s educational inactivity. As a result, in 15 weeks the student achieves the standard “excellent” grade because he/she manages to forget some course details in the last 7 weeks of course dropping.

Fig. 11 shows two peaks of student interest in the course: the first lower peak of student interest takes place during the first week of the course and the second higher peak of student interest corresponds to the 9<sup>th</sup> week of a 15-week-long course.

Figs. 8–11 show course-learning approach “E” of an ambitious but simultaneously working student who does their best to apply the maximum learning efforts in the first midterm time of course acquisition and completely drops out in the second midterm of course duration.

X. STUDENT COURSE-LEARNING STRATEGY F “STARTING HERO” OR “SPRINTING HERO”: INTENSIVE START LEARNING ACTIVITY

Student intensive start learning strategy “F” is outlined in Figs. 12–17, which corresponds to the integral performance indices (14)–(19) of student learning activity.

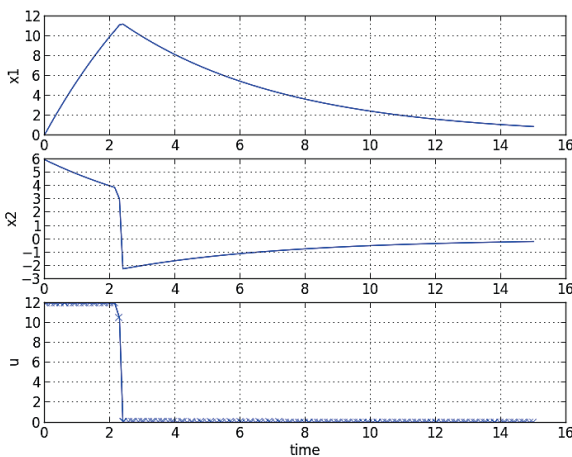


Fig. 12. Educational strategy No.12 of the learning behaviour of an academically diligent and hardworking student who studies extremely intensively only during the first 2.5 weeks of the semester and then rests for the next 12.5 weeks of the academic semester, which remain until the examination session.

Fig. 12 shows the twelfth course-learning strategy with intensive start learning activity of a hard-working student which is determined by the optimal control problem (1)–(2) with the minimized objective function (14), where the twelfth

minimized integral functional (F1) is defined as a minimum of a definite integral of the difference of the control signal minus the first phase variable over the semester-long time interval during which the course-enrolled student acquires the new discipline, i.e.,  $\min(J_{12}) = \min(\int(0 \text{ to } T) (u(t) - (x_1(t))) dt)$ .

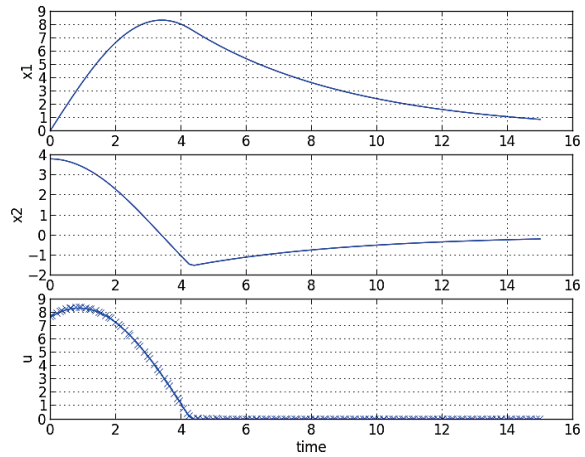


Fig. 13. Educational strategy No.13 of the learning behaviour of an academically diligent and hardworking student who studies extremely intensively only during the first 3.5 weeks of the semester and then rests for the next 11.5 weeks of the academic semester, which remain until the examination session.

Fig. 13 shows the thirteenth course-learning strategy with intensive start learning activity of a hard-working student which is determined by the optimal control problem (1)–(2) with the minimized objective function (15), where the thirteenth minimized integral functional (F2) is defined as a minimum of a definite integral of the difference between the square of the control signal minus the square of the first phase variable over the time interval during which the student studies the new course, i.e.,  $\min(J_{13}) = \min(\int(0 \text{ to } T) (((u(t))^2) - ((x_1(t))^2)) dt)$ .

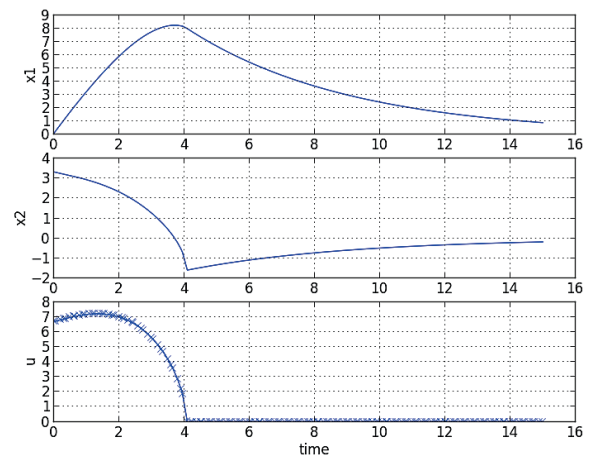


Fig. 14. Educational strategy No.14 of the learning behaviour of an academically diligent and hardworking student who studies extremely intensively only during the first 4 weeks of the semester and then rests for the next 11 weeks of the academic semester, which remain until the examination session.

Fig. 14 shows the fourteenth course-learning strategy with intensive start learning activity of a hard-working student which is determined by the optimal control problem (1)–(2) with the

minimized objective function (16), where the fourteenth minimized integral functional (F3) is defined as a minimum of a definite integral of the difference between the cube of the control signal minus the cube of the first phase variable minus the cube of the second auxiliary variable over the semester-long time interval during which the course-enrolled student acquires the new discipline, i.e.,  $\min(J_{14}) = \min(\int(0 \text{ to } T) (((u(t))^3) - ((x_1(t))^3) - ((x_2(t))^3))dt)$ .

Fig. 15 shows the fifteenth course-learning strategy with intensive start learning activity of a hard-working student which is determined by the optimal control problem (1)–(2) with the minimized objective function (17), where the fifteenth minimized integral functional (F4) is defined as a minimum of a definite integral of the product of the sixth power of the first time derivative of the first phase variable, multiplied by the difference of the cube of the first phase variable minus the control signal, multiplied by the difference of the cube of the second auxiliary variable minus the control signal over the semester-long time interval during which the student acquires the new course, i.e.,  $\min(J_{15}) = \min(\int(0 \text{ to } T) (((x_1(t))^3 - u(t)) \times ((x_2(t))^3 - u(t)) \times (d/dt(x_1(t)))^6)dt)$ .

Fig. 16 shows the sixteenth course-learning strategy with intensive start learning activity of a hard-working student which is determined by the optimal control problem (1)–(2) with the minimized objective function (18), where the sixteenth minimized integral functional (F5) is defined as a minimum of a definite integral of the product of minus one half, multiplied by the sum of the square of the first phase variable with the addition of the square of the second auxiliary variable with the addition of the square of the control signal over the semester-long time interval during which the course-enrolled engineering student acquires the new {undergraduate, graduate}-level discipline, i.e.,  $\min(J_{16}) = \min(\int(0 \text{ to } T) ((-1) \times (1/2) \times (((x_1(t))^2) + ((x_2(t))^2) + ((u(t))^2)))dt)$ .

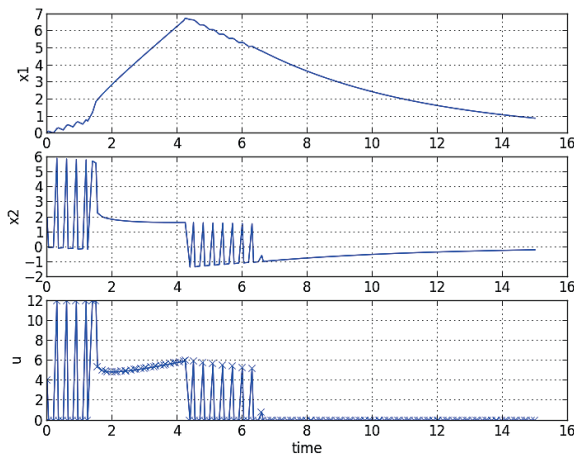


Fig. 15. Educational strategy No.15 of the learning behaviour of an academically diligent and hardworking student who studies extremely intensively only during the first 4.5 weeks of the semester and then rests for the next 10.5 weeks of the academic semester, which remain until the examination session.

Fig. 17 shows the seventeenth course-learning strategy with intensive start learning activity of a hard-working student which is determined by the optimal control problem (1)–(2) with the

minimized objective function (19), where the seventeenth minimized integral functional (F6) is defined as a minimum of a definite integral of the product of minus one half, multiplied by the sum of one with the addition of the square of the control signal over the time interval during which the student studies the new course, i.e.,  $\min(J_{17}) = \min(\int(0 \text{ to } T) ((-1) \times (1/2) \times (((u(t))^2) + 1))dt)$ .

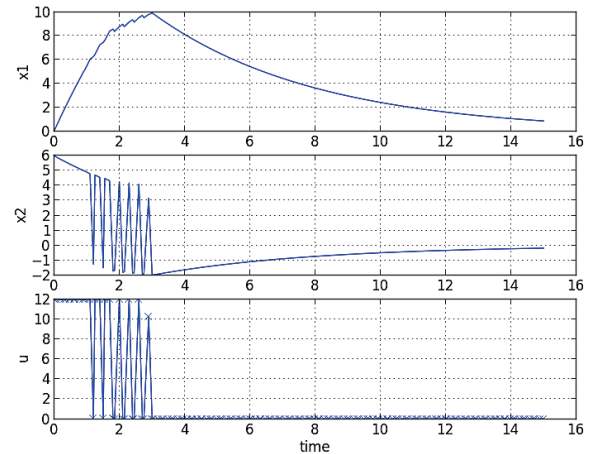


Fig. 16. Educational strategy No.16 of the learning behaviour of an academically diligent and hardworking student who studies extremely intensively only during the first 3 weeks of the semester and then rests for the next 12 weeks of the academic semester, which remain until the examination session.

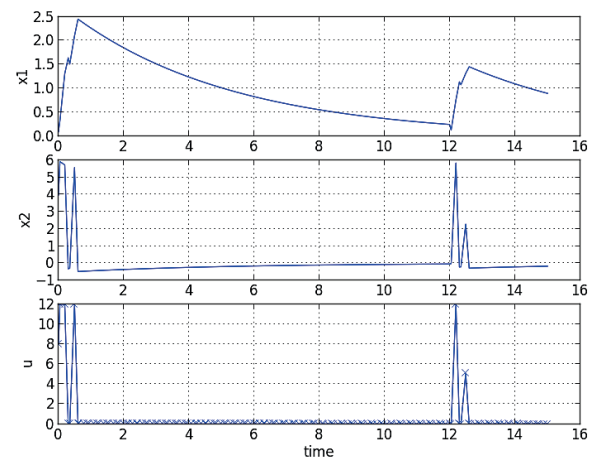


Fig. 17. Educational strategy No.17 of the learning behaviour of an academically diligent and hardworking student who studies extremely intensively mainly during the first and thirteenth weeks of the semester and then rests for the rest of the 13 weeks of the academic semester that remain until the examination session.

Figs. 12–17 show that the intensively-working student with hyperactive initial performance successfully completes all mandatory and facultative course-required assignments in the first 2.5 weeks (Fig. 12), in the first 3 weeks (Figs. 13 and 16), in the first 3.5 weeks (Figs. 13 and 14) or in the first 4.5 weeks (Fig. 15). Later, after an intensive start, the student completely drops out from further course learning and does nothing for the remaining 12.5 weeks (Fig. 12), for the remaining 12 weeks (Figs. 13 and 16), for the remaining 11.5 weeks (Figs. 13 and 14) or for the remaining 10.5 weeks (Fig. 15).

Fig. 17 shows two peaks of student interest in the course: the first higher peak of student learning interest takes place during the first week of the course, and the second lower peak of student learning interest takes place after 12.5 weeks of a 15-week-long course.

Figs. 12–17 show a rather extreme course-learning approach “F” of initially-hyperactive learner who makes their best to apply all available learning efforts in the first 2.5...4.5 weeks of course acquisition and completely drops out from the further course participation in the remaining 12.5...10.5 weeks of the course. The intensive start learning approach is an effective learning strategy for experienced professionals who are ready for quick educational immersion into a new course.

## XI. DISCUSSION

The following six student course-learning  $J$ -dependent educational strategies were  $J$ -formulated, computationally determined, and graphically visualized: (A) “Lazy Student” with a late final learning activity, (B) “Procrastinator” with a delayed learning activity, (C) “Growing Student” with a monotonic growing learning activity, (D) “Steady Student” with an ongoing learning efforts activity, (E) “Midterm Hero” (or “Halfway Hero”) with a maximum midterm learning activity, and (F) “Starting Hero” (or “Sprinting Hero”) with an intensive start learning activity.

Strategy A “Lazy Student” can be called “The big lazy man” because strategy A describes the behaviour of a student who studies a semester course for the last three days.

Strategy B “Procrastinator” can be called “The procrastinator” because strategy B describes the behaviour of a student who only starts studying in the second half of the semester.

Strategy C “Growing Student” can be called “A student with a monotonically increasing level of knowledge” because strategy C describes the behaviour of a student who monotonously studies the course throughout the semester.

Strategy D “Steady Student” can be called “A student with a pulsating stable level of knowledge” because strategy D describes the behaviour of a student who has studied enough material at the beginning of the course to get an excellent grade and then regularly maintains their level of knowledge.

Strategy E “Midterm Hero” or “Halfway Hero” can be called “A student who runs the first half of the distance and then walks the second half of the distance” because strategy E describes the behaviour of a student who reaches their peak study activity approximately in the middle of the academic semester.

Strategy F “Starting Hero” or “Sprinting Hero” can be called “A student who sprints the starting quarter of the distance and then walks the final three-quarters of the distance” since strategy F describes the behaviour of a student who reaches their peak academic activity in the first third of the academic semester and then mostly rests in the remaining two-thirds of the semester.

Computational plots in Figs. 1–17 outline that learning sustainability is rather different for engineering students and working professionals. The learning trends “B” (Figs. 3 and 4),

“C” (Figs. 5 and 6), “D” (Fig. 7), and “E” (Figs. 8–11) are equally possible for both students and engineers. However, the learning trend “F” (Figs. 12–17) is better suited only for prompt learning of graduate engineering students and continuing professional development of the working professionals who are psychologically ready to a highly intensive learning rate with ultra-fast pace of initial learning. The learning trend “A” (Figs. 1 and 2) is the most unsustainable, undesirable and highly-irritating learning strategy, which is not recommended for both students and engineers.

In the work [42], considerable attention is paid to the analysis of the complex and implicit dynamic relationship of superficial, deep and transitional stages in multi-level educational strategies of teaching students, taking into account the need to simultaneously ensure a high level of student excitement when studying new educational material “with passion” and in competition with classmates [42]. If we use the generally accepted pedagogical terminology about “superficial” and “deep” educational strategies with passion or without enthusiasm, then we can assume that Figs. 1–6 in the presented work still correspond more to superficial learning strategies without any particular enthusiasm on the part of the student, and Figs. 7–17 in the article, to a greater extent, correspond to deep educational strategies of fairly enthusiastic student learning “with passion”.

It should be noted that the present theoretical study (Figs. 1–17) may still be of some academic interest to experimental researchers due to the fact that this work only deals with six theoretically established strategies (A–F) of student learning, whereas the empirical work [42] very briefly mentions more than 400 experimentally established strategies of student learning, which is more than 67 times the number of six theoretically calculated optimal student learning strategies presented in this computational study.

It is interesting to note that the educational study of the nonlinear optimal control problem (1) is also required in engineering curriculum [43] because computational research into learning dynamics of a forgetful student was proposed as individual homework exercise number 3 to chapter 13, p. 754 of the robotics textbook [43]. Thus, the author-derived results of nonlinear computational modelling of student learning dynamics (Table 1, Figs. 1–17, (1)–(19), dataset [44]) may be of interest not only to specialists in social dynamics and psychological and pedagogical modelling, but also, to some extent, help future students of engineering specialties better understand specialised technical literature on optimal control in robotics [43].

## XII. CONCLUSION

The paper is focused on the effect of performance index selection on student learning dynamics.

Computational research into phenomenological dynamics of student learning is very important for engineering students, working engineers, managers and team leaders because today all members of engineering and R & D communities stay under

ongoing severe administrative pressure to efficiently acquire new technical knowledge in a timely way.

The author's computational approach substantially broadens student ideas about nonlinear impact of student  $J$ -performance index on the educational peculiarities of u-guided student learning.

Computationally found student learning trends are suitable for phenomenological description of production and heutagogical activities of a working engineer during timely project completion, continuing professional development and lifelong professional learning.

The author's computational approach (Table 1, Figs. 1–17, (1)–(19), dataset [44]) has found successful instructional implementation in the engineering curriculum for undergraduate, graduate and PhD students as well as in continuing professional development of researchers and technical instructors of Donbas State Engineering Academy, Kramatorsk, Ukraine.

#### *A. Limitations of the Present Research*

The author's approach is characterised by a number of gross simplifications and obvious limitations that significantly reduce the generality of the results obtained in the research.

L1. The idealized socio-cybernetic formulation of the optimisation problem of the pedagogical and psychological dynamics of nonlinearly controlled student learning in the form of a boundary value optimal control problem is based on a very naive and completely unfounded assumption of successful and timely study of a new course by a student within the framework of the implementation of a  $J$ -dependent learning strategy along some  $J$ -specified trajectory during a fixed time of studying a new discipline  $T$  (week). For example, during the duration of a university semester, it completely ignores such real limiting circumstances as the constant lack of time, motivation, computing resources and academic qualifications of a typical academically inactive student, which in most real cases makes methodologically meaningless both the very promising conversation about a fixed planned time of successful study of a new course by an uninterested student along some completely special and, apparently, extremely successful individually selected  $J$ -trajectory of deeply personalized dual learning, and nullifies any pedagogical significance of the computational solution of the corresponding nonlinear boundary value problems of optimal control of the student's learning dynamics depending on the time parameter. It happens since the timely mastering of a new course of study is unfortunately either a naively idealized assumption or a formally bureaucratic pedagogical concept, practically used by all educational institutions whose main task is exclusively the issuance of a certificate or diploma for the "successful" completion of training, regardless of the actual successes or failures of a student officially enrolled and formally studying at the educational institution.

L2. The only source of the emergence of computational nonlinearities in the considered boundary value problem of optimal control of the educational dynamics of student learning with a  $J$ -variable individual student choice of one or another

nonlinear  $J$ -educational strategy of individual student acquisition of a new course is determined exclusively by the {Modelica, Optimica}-solvable nonlinear algebraic structure of the nonlinear objective function  $J$ . It additionally limits the nonlinear optimisation problem and, for one reason or another, is constructed by the author of the study in the form of a pedagogically admissible minimized  $J$ -functional, which actually determines a certain individual nonlinear educational strategy of studying a new discipline by a student. The differential equation of the dynamics of student learning itself is an ordinary linear differential equation of the first order, which does not contain either time derivatives of higher orders from the phase variable, or time derivatives of the control signal, or higher powers of the phase variable and the control signal, or their cross products, or such nonlinear system-dynamic components as stagnation zone, saturation zone, damping effects, etc.

L3. The main computational emphasis of the study was placed on the engineering and mathematical design of didactically possible and {Modelica, Optimica}-solvable  $J$ -strategies of pedagogically permissible nonlinear strategies of optimally controlled study of a new course by a student with subsequent possible classification of Modelica-obtained optimisation solutions into certain generalized trends of student behaviour based on the geometric similarity of  $J$ -defined graphical constructions, as well as based on certain psychological and pedagogical considerations. While an extended nonlinear-dynamic analysis of the possible presence of additional bifurcation effects in each of the considered  $J$ -strategies of student learning still goes far beyond the scope of the present study, which focused exclusively on the rational engineering  $J$ -design of a certain trajectory of study of a new course by a student with subsequent pedagogical classification of the obtained  $J$ -learning trajectories into certain behavioural trends of individual student study of a new academic discipline.

#### *B. Opportunities for Future Research Areas*

P1. In the further research on the local dynamics of student learning of a new course, the author plans to devote more attention to practical issues related to the direct determination of approximate numerical estimates of minimized functionals within the framework of an applied comparative analysis of both various student goal-settings and various student learning strategies depending on the didactic complexity of the new course, the student's age, the degree of personal motivation to study the new course, etc. It would be interesting to see how the considered index-dependent student learning is applied to particular study courses and programs. This could include some objective (numerical) evaluation of learning achievements, as well as feedback from involved (students, university staff, employers, etc.).

P2. In this paper, each student's goal setting was considered as an individual optimal control problem with a specific mathematical construct of the nonlinear objective function. In the future, for each student's goal setting, it would be important to mathematically investigate the theoretical issues of solution existence, solution uniqueness, and actual solution optimality,

as well as to conduct additional research into the very possibility of nonlinear dynamic phenomena such as bifurcation effects.

P3. Based on constructivist considerations, the author of this study suggests that there is a non-obvious but objective relationship between the mathematical construct of the minimized functional proposed by the author and a student's specific behavioural goal-setting when studying a new course. Furthermore, the author even suggests that each mathematical formula for a nonlinear objective function defines psychological and pedagogical patterns of student learning behaviour when studying a new course. Furthermore, each author's assumption about the existence of a particular relationship between the proposed mathematical formulas for the optimisation description of learning goal-setting and the corresponding types of individual student behaviour apparently requires not only further substantiation but also further complex proof.

P4. The presented approach requires further replication and the broadest possible testing, involving numerous additional recalculations of the author's dataset within the context of individual student computational assignments with varying numerical parameters for each series of optimal control problems involving student learning dynamics. Only through the generalization of a large-scale numerical experiment will it be possible to draw further, more comprehensive conclusions about the nature of the nonlinear influence of various numerical parameters of the optimisation problem of rational student goal-setting when studying a new course on the type of learning strategy. Even if extensive numerical modelling reveals that, for certain numerical parameters of the optimisation problem, a particular goal-setting approach will lead to a numerical solution whose graphical representation does not correspond to the trend proposed by the author, it is likely that such an optimisation problem will still fall within one of the six student learning strategies proposed by the author within the framework of this engineering and pedagogical study.

P5. In the context of further computational research, it will be possible to further complicate the fundamental differential equation of student learning by introducing a larger number of terms and further analyse how exactly the numerical solution for each of the goal settings will change and clarify whether some redistribution of goal settings will occur within the framework of the formulated strategies for student learning of a new course.

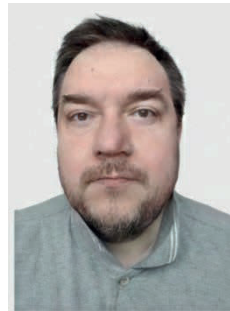
P6. As part of the organisation of further experimental research, it is planned to conduct anonymous student surveys to determine the greatest popularity of one or another educational strategy for studying a new course, depending on the topic of a new academic discipline, the age of the student, individual motivation, and a wide range of other socio-psychological parameters.

P7. The considered educational behavioural strategies of student learning (A–F) can be further analysed using such modern theoretical approaches as methods of game theory, agent-based modelling, graph and network theory, as well as structural equation modelling methods.

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