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Price of Water

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Abstract **– There are watercourses on the globe which as yet do not deliver up their energy to the needs of the people. How much energy their waters bear, is it worth to take away this energy? Those and alike questions must be (and they are) answered before start to build hydro power station. Similar problems must be solved to control hydro power plants in most gainful way which is known as hydrothermal coordination. The notion of price of water can be met lately in technical literature as one of numerical indices of these issues. The gross price of water and net price of water are considered in this paper. Gross price of 1 t water is the price of electric energy obtained by conversion of potential energy of 1 t of water, lifted to a height of power station water head. Net price of water is the difference between gross price and total expenses determined by hydro power station building and its exploitation costs in a year related to 1 m^3 of water. If net price of water is positive, it is worth building power station. The greater net price is, the more urgent is the building. Net price of water grows with water head but it continues only to some height of the dam because further increase of head sharply increases capital outlay and other exploitation expenses. To maximize net price of water, optimization of net price function can be done. Net price of water diminishes when some amount of water is diverted for other needs. When amount of diverted water is out of discussion, no controversy can emerge. However when by diverted water some goods with some monetary worth can be obtained, the task must be solved how much water can be diverted so that the water of watercourse be used to the maximum benefit. The environmental issues must be taken into account as well.**

Key wards **– hydro power plant, hydrothermal coordination, price of electricity, short-term generation scheduling, value of water, water dam**

I. INTRODUCTION

There are many rivers, rivulets, brooks and streams. As yet, not all of them serve to the benefit of humans, even in Europe. The problem stands: whether it is purposeful to build hydropower plant on these watercourses. Is it better to do without electrical generators which use water energy at hand and buy all the necessary electricity from utility or from abroad or is it better to build a power plant of appropriate capacity according to corresponding water flow at place? This alternative isn't new and no any power station is started to erect without answering this question. Before to start building, feasibility study should be made.

On the other hand, hydropower plant building completed, the mode of it's mostly economic operation must be pursued permanently (operation scheduling). This task is known as hydrothermal coordination solved daily by utilities' dispatchers. The ways of solving this questions are broadly discussed in technical literature [1]. In ref. [2] A.J. Pansini and K.D. Smalling write: "The generation of hydro units in a system in which both hydro and thermal generation are used presents an extension of the economic loading problem. However, if a value (do read: price) can be placed on water in each reservoir, usually in dollars per acre foot, hydro units can be operated incrementally along with thermal units for overall economic operation of the system".

This phrases in plain and unambiguous manner suggest the notion of the price of water. Sometimes it must be contended whether to produce electricity or use water for other non-less significant purposes. Price of water changes in dependence on circumstances. It is lower when alternative sources are redundant or during periods of high flows. Since each cubic meter of water can produce some amount of electrical energy, water can substitute for any other source of energy, this being particularly significant for fossil fuels. This effect is known to all specialists which have to do with the control of power station operation. However word "price" too much resembles the market place of goods from one side. From the other side, water is tribute of nature and as such it is gratis.

To do away with these scruples, let us consider the mushrooms. They are not sown nor cultivated yet they are in abundance in a wood at a proper place and time of the year. They are the gift of nature as well as water in a river. However, if we go to market place and buy them, we must get rid of some money. But when we go to the forest and pick them to a basket, our money rests untouched and when we realize some gathered amount, we become richer by some value of money and it had happened since mushrooms have market price notwithstanding that they are gratis in terrain.

The price of water is well known in arid regions of the globe, being for instance, in Arab Emirates more costly than petrol. Urban population is aware too good what it is the price of water. But as far as in a river water flows freely and its amount is incomparable with household needs, the price of water is out of any practical meaning when it is not used for electricity generation.

Formation of prices and the best prices on competitive basis is considered in the article.

II. GROSS PRICE OF WATER

At the beginning we must remind the well known fact that potential energy of weight G of water or any other substance raised to height H is

$$A_p = GH \qquad (1)$$

The equivalent value of potential mechanical energy and electrical energy can be found using the ratios between corresponding mechanical and electrical units [3]. Using international unit system IS, the basic ratio between 1 ton meter of mechanical energy and 1 kWh of electrical energy as well as ratio between corresponding power are:

1 tm = 2,725 Wh = 0,002725 kWh ; 1 tm/s = 9,81 kW (2)

At first two points must be stated. Present consideration is made supposing that: 1) owner of generating facility is power system; 2) price of electricity *c* or price of any goods acquired thanks to water does not depend on the amount of electricity sold or purchased by said power system.

The commonplace notion of price of electric energy is different at various levels of finance calculus, and what is yet more, the price varies in time short-term (hourly and daily) and long-term (yearly usually upward). If one kWh costs *c* monetary units, then 1 ton meter, converted to electricity according with (2), theoretically costs 0,002725*c*. It would be precisely so if entire potential energy would be converted to electricity but it never uses to be and efficiency factor η must be taken into account.

After unofficial information at Riga HES one hydro generator produces 67 MW power consuming 700 cubic meter water per second at water head of 12 m. The power of this water discharge, observing that 1 m^3 of water weighs 1 t, is P_w = 12x700 = 8 400 tm/s which, according with (2), is P_e = 8400 x 9,81 = 82, 4 MW. These figures give sought for efficiency factor η = 67 / 82,4 = 0,813.

Efficiency factor depends on water head and slightly grows with the water head, on height H it is η_H. Besides, price of electricity *c* is not constant in time, for instance, in congested hours when hydro power usually is used for electricity generation the price *c* is much higher then at night, hence the effective value of price of electricity c_e which is higher than mean value must be taken in our consideration. Then price of 1 t of water lifted to the height of 1 m will be:

$$c_H = 0{,}002725\eta_H c_e \tag{3}$$

One ton of water raised at height *H* have energy *H* tm, hence gross price C_H of one ton of water, in other words, annual effective value of gross water price, is:

$$C_H = c_H H \tag{4}$$

This quantity takes into account no additional expenses which are indispensable for electricity generation.

III. NET PRICE OF WATER

Gross price of water is of significance for big hydropower stations when pay-back period is expired and operation costs are not great. If we consider all expenses connected with pay-back, exploitation and maintenance, then we must consider all positions of buildup and running expanses of the object. As a time unit for integral indices we take one year. The yearly gross income I_y of hydro power station can be determined as:

$$I_y = C_H F_y, \tag{5}$$

where F_y – integral flow of water during a year.

The real (net) income in a year must take into account all the related expenses E_y:

$$I_{ynet} = I_y - E_y \tag{6}$$

Net price of water equals annual net income divided by annual cubic meter number F_y of water which flows through the turbines of the plant:

$$C_{Hnet} = I_{ynet} / F_y \tag{7}$$

As annual expenses E_y, in reality there are a lot of positions which all contribute to decrease the net annual income. We shall use three enlarged terms:

$$E_y = E_c + E' + E'' \tag{8}$$

The first term takes into account capital outlay *K* and constant expanses E_{op} insuring power plant operation [4]:

$$E_c = (i + p_\Sigma)K + E_{op} = f(H), \tag{9}$$

where *i* – interest rate; p_Σ –summary deductions which vary with the years of exploitation. To simplify considerations, it is advisable to determine *i* and p_Σ assuming the pay back period value near to life cycle and to considering it as constant values. E_{op} – constant operation expenses in the sense that they do not depend on amount of generated energy. Both terms of (9) grow with dam height and its length, hence they are function *f(H)* of water head *H*. The second term E' of (8) takes into account expenses proportional (proportionality factor E_{op}') to amount of generated energy; they take into account no load losses of equipment, which capacity on duty grows with generated power, and other expenses, e.g. salaries of operational personal:

$$E' = E_{op}' I_y \tag{10}$$

The third term E'' takes into account mainly load losses which are proportional to square of current, hence to squared annual gross income (with the rate E_{op}''):

$$E'' = E_{op}'' I_y^2; \tag{11}$$

This term can have notable value if power plant is situated at considerable distance from energy consumer.

Then from (6) – (11) the net annual income I_{ynet} will be:

$$I_{ynet} = (1 - E_{op}')I_y - E_{op}'' I_y^2 - f(H). \tag{12}$$

Observing (4) – (12), we have:

$$C_{Hnet} = (1 - E_{op}{}')c_H H - E_{op}{}'' c_H{}^2 H^2 F_y - \frac{1}{F_y} f(H) \quad (13)$$

Regarding net price of water, the decision can be made whether it is worth building hydropower plant. If net price of water is negative, the building of power station is not profitable. There can be no owner of power station that would generate electricity with expenses higher than income from electricity generation.

It is expedient to take out of watercourse as much energy as is possible taking into account economic and environmental reasons. One can reckon on dam of some height and become negative price of water and resign building. While the other increase the height of dam and it turns out to be useful. Hence (13) must be optimized as a function of height H. It is obviously that the greater is water head, the higher is gross price of water C_H. But it in no way means that net price of water will raise. The first term of (13) will grow but so will do the second and mainly third term.

Derivative of net water price with respect to water head:

$$\frac{dC_{Hnet}}{dH} = c_H(1 - E_{op}{}') - 2E_{op}{}'' c_H{}^2 F_y H - \frac{1}{F_y}\frac{df(H)}{dH} \quad (14)$$

At some height of dam H, derivative will be zero and there will be water price maximum. It can be proven if at least at two values of height H price is zero (C_{Hnet}=0). But namely this in reality can take place: when water head is zero ($H=0$), price is negative since $f(H)\neq 0$, when H grows above admissible limits, the third negative term of (13) tends to infinity and price becomes negative again (Fig. 1). Since derivative dC_{Hnet}/dH somewhere between those points is zero and beyond those points C_{Hnet} is negative then somewhere between those H values water price is positive. Hence the price will be maximum at some places where derivative is zero. If $df^2(H)/dH^2$ never diminishes with H, it is, if costs of building power plant grow accelerating with dam height, then maximum price of water is at only one definite height of dam.

In such a way the optimum dam height H can be determined using water price notion.

IV. COMPETITIVE ASPECT OF WATER PRICE

We have considered amount of water flow F_y as a given value. But what it is in reality? It depends on what natural circumstances are.

In Egypt, for example, precipitations are so scarce that water flow of river Nile can be considered as inflow out of East Equatorial Africa. The Aswan High Dam, [5], is 3,830 m in length, 980 m wide at the base, 40 m wide at the crest and 111 m tall. At maximum, 11,000 m³ of water can pass through the dam every second. The dam powers twelve generators each rated at 175 megawatts, producing a hydroelectric output of 2.1 gigawatts. The reservoir stores 111 km³ of water. Egypt's agriculture depends entirely on irrigation. With irrigation, nearly two crops per year can be produced. Aswan Dam releases on average F_y = 55 billion m³ water per year of which some F_{yd} = 46 billion m³ are diverted into the irrigation canals. In the Nile valley and delta, approximately 3,36 million ha benefit from these waters producing on average 1.8 crop per year. Now we see that almost 84 % of water flow is diverted to agricultural needs.

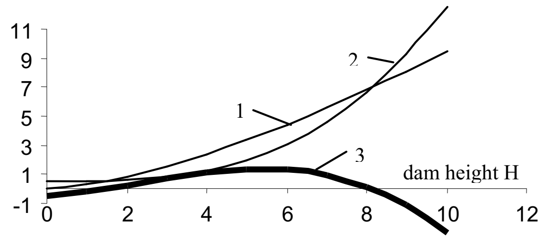

Fig. 1. Optimization of net price of water with respect to dam height

1 – gross price; 2 – specific expanses; 3 – net price

In the temperate climate water flow is determined mainly by precipitation conditions; although near big cities, considerable amount of water F_{yd} is diverted to the needs of urban population. If the amount of diverted water F_{yd} is out of discussion because it is indispensable for inhabitants or for other purposes (e.g. for industry) which allow no options as to amount of water necessary, then the water flow for electricity generation F_{ye} is diminished:

$$F_{ye} = F_y - F_{yd} \quad (15)$$

and net water price for electricity generation diminishes since the third term of (16) sharply grows:

$$C_{Hnet} = c_H(1 - E_{op}{}')H - \\ - E_{op}{}'' c_H{}^2 (F_y - F_{yd}) H^2 - \frac{1}{F_y - F_{yd}} f(H) \quad (16)$$

If amount of diverted water can be discussed on the economical basis, then we can see competitive character of two categories of prices: the price of water for electricity generation C_{Hnet} and the price of diverted water C_d. The price of diverted water can be determined by the cost of measures which substitute diverted water. Using some amount of diverted water F_{yd}, the crop of cereals can be grown and it would be unnecessary to buy this cereals from abroad for market prices. If this amount of water is passed through the turbines of power plants, some amount of energy will be generated and it would be unnecessary to buy it from abroad. What measure saves more money?

The reasonable sense (if other views and opinions are brushed away) prompts that the best solution is when total income out of electricity production and crop growing is maximum:

$$I_{y\Sigma} = C_{Hnet}(F_y - F_{yd}) + C_d F_{yd} = \max \quad (17)$$

Extended on the base of (16) and contracted, (17) looks out as:

$$I_{y\Sigma} = c + bx - ax^2 = \max\ , \quad (18)$$

where

$$x = F_{yd}\ ;\ a = E_{op}{}'' c_H{}^2 H^2\ ;\ b = C_d + 2aF_y - C_H{}'\ ;$$
$$c = (C_H{}' - aF_y)F_y - f(H)\ ;\ C_H{}' = c_H(1 - E_{op}{}')H \quad (19)$$

The extremum of function (18) will be at

$$x_e = b/2a \quad (20)$$

To seek for maximum of function (18), we note, that if $a>0$ and $b>0$, expression $bx - ax^2$ has two non negative roots: $x_1=0$; $x_2=b/a$. It means that straight line bx and curve ax^2 intersect at points x_1 and x_2. Beyond these points expression $bx - ax^2$ is negative, than between points x_1 and x_2 $bx - ax^2$ must be positive and extremum of function (18) is a maximum i.e. the optimum value of diverted amount of water is $b/2a$.

So far it is good. But what will be if $b<0$? Maximum income (18) will be at negative diverted amount of water F_{yd} which is senseless and entire flow of water must be passed through turbines of power plant.

And what will be at special case when load losses are negligible, i.e. when $a=0$? In such a case we have annual income:

$$I_{y\Sigma} = C_H{}' F_y - f(H) + (C_d - C_H{}')F_{yd} \quad (21)$$

This function has no extremum. It means that if $C_H{}' > C_d$, all water must be passed through turbines, otherwise – to crop growing. Only when $C_H{}' = C_d$ it is all the same how to utilize water.

Such situation when $a=0$ is unlikely in practice. It can never happen that prices $C_H{}'$ and C_d are equal. In arid regions, some amount of water is let to power plant and remaining amount – to other needs. It means that coefficient a never equals zero. Some amount of diverted water will be necessary to such an extent that its price sharply grows. It is equivalent to that losses ax^2 with growing amount of electric energy produced sharply grow. Strictly speaking, term ax^2 in (18) must be substituted by some function $f(x)$ and (18) will transform to:

$$I_{y\Sigma} = c + bx - F(x) = \max \quad (22)$$

The function $F(x)$ is increasing and concave hence optimum diverted amount of water exists. As concrete case of this assertion are flood waters when power plant either can not utilize entire amount of water or generated electricity can not be consumed and the surplus of water is to be used in some other way. If this is not done, the remnant of water (not used water) can be considered as losses. Environmental issues also add to this reasoning.

The consecutive optimization of dam height, using (14), and optimization of amount of diverted water (using (18) – (20)) can lead to optimum watercourse utilization.

As we have seen, in Egypt optimum water passed through hydropower plant is only 16 %. The ratio of water prices is such that maximum benefit is when $F_{yd}\%$=84 and only 16 % is used for electricity generation. What a situation will be when population of this country still more increases, is hard to visualize.

The other example of water utilization is Niagara river. In 1950, the United States and Canada signed the Niagara River Water Diversion Treaty [6]. "Its purpose is to preserve the scenic beauty of Niagara Falls…". Under this treaty, flow of water over Niagara Falls should be maintained not less than 2830 m^3/s during the specified day time spans between specified dates of the year or not less than 1415 m^3/s at any other time. During these specified day time spans, the beauty of the waterfall is available for tourists. The energy of the water, diverted for surprise and delight of the humans, must be worked off by burning of fossil fuels. The 1 m^3 price of this water then, in short, equals to the price of equivalent amount of fossil fuels. To what extent this loss can be reimbursed by tourist fee it is the question of bookkeeping but it in no way questions the notion of water price.

But what financial loss tolerates the power company when it is forced to pass the water over the dam during spring floods? It is easy to calculate, knowing the amount of discharged water and its price.

V. CONCLUSIONS

1. Gross price of water is determined as cost of electrical energy which can be produced by potential of 1 cubic meter at specified head of water.
2. Net price of water is income out of electrical energy which can be produced by potential of 1 cubic meter of water observing all expenses necessary for electricity generation.
3. The portion of water evading electric generator diminishes net price of water.
4. Net price of water can be criterion for building hydro power plant as well as to control electricity generation.

VI. REFERENCES

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