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A Sensorless Speed Control System for DC Motor Drives

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***Abstract* - An approach to sensorless speed control of permanent magnet DC motor drives is presented in this paper. The motor speed has been estimated indirectly by the respective back EMF voltage. Using a discrete vector-matrix description of the controlled object, an optimal modal state observer has been synthesized, as well as an optimal modal controller. The results obtained show that the applied control method can ensure good performance.**

I. INTRODUCTION

The sensorless speed control is attractive for many technical applications, because it reduces hardware costs and improves mechanical reliability. For this reason development of driving systems without sensors for the respective mechanical coordinates is a topical problem of modern electric drives theory [1].

An approach to sensorless speed control of DC motor drives is discussed in this paper. The controlled object is an electromechanical system which consists of a four-quadrant transistor chopper and a permanent magnet DC motor. In this case motor speed has been estimated indirectly by the respective back EMF voltage [2].

Using a discrete vector-matrix description of the controlled object, a state observer has been synthesized, as well as the respective optimal modal controller applying a complex criterion for optimization [3].

Detailed study has been carried out by means of mathematical modeling and computer simulation for the dynamic and static regimes at various loading conditions and disturbances. The results presented show that the applied method of control can provide for good performance.

II. MODELING OF THE CONTROLLED OBJECT

The vector-matrix model of the controlled DC motor drive is as follows:

$$\begin{bmatrix} \dfrac{dE}{dt} \\ \dfrac{di_a}{dt} \end{bmatrix} = \begin{bmatrix} 0 & \dfrac{K_e K_t}{J} \\ -\dfrac{1}{L_a} & -\dfrac{R_a}{L_a} \end{bmatrix} \begin{bmatrix} E \\ i_a \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{K_c}{L_a} \end{bmatrix} v + \begin{bmatrix} -\dfrac{K_e K_t}{J} \\ 0 \end{bmatrix} i_l , \quad (1)$$

where: E is back EMF voltage; i_a - armature current of the motor; K_e - back EMF voltage coefficient; K_t - torque coefficient; R_a - armature circuit resistance; L_a - armature inductance; K_c - amplifier gain of the chopper; v - input control signal of the chopper; J - total inertia; i_l - armature current which is determined by the respective load torque.

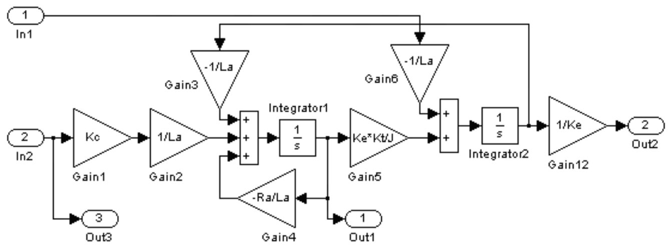

Fig. 1. Block diagram of the controlled DC motor drive.

The basic parameters of the controlled object are as follows: $K_e = 0.191$ Vs/rad ; $K_t = 0.191$ Nm/A ; $R_a = 0.61\,\Omega$; $L_a = 0.003$ H ; $K_c = 3.16$; $J = 0.0043$ kg.m^2 .

The rated data of the used permanent magnet DC motor are: $V_{rat} = 30$ V, $I_{rat} = 15.7$ A, $\omega_{rat} = 115.19$ rad/s .

The analogue model of the DC electric drive is realized according to (1) and it is represented in Fig. 1.

Because the motor velocity is not measured directly, the EMF back voltage $E = K_e\omega$ is used, which can be calculated in the following way:

$$E = v_a - R_a i_a - L_a \frac{di_a}{dt} . \quad (2)$$

For small quantization periods T (2) can be transformed into the next equation:

$$E(k) = v_a(k) - [\,R_a i_a(k) + \frac{L_a}{T} i_a(k) - \frac{L_a}{T} i_a(k-1)] \quad (3)$$

Based on the (3) a discrete model of the back EMF voltage has been developed which is shown in Fig. 2.

The following notations of state variables have been adopted:

$$x_1 = E\ ,\ x_2 = i_a\ .$$

The measurable coordinate in this case is the motor back EMF voltage E, i.e.

$$y(t) = \mathbf{C}\mathbf{x}(t),$$

where $\mathbf{C} = [1 \quad 0]$, and $\mathbf{x}^{\mathrm{T}} = [x_1 \quad x_2]$.

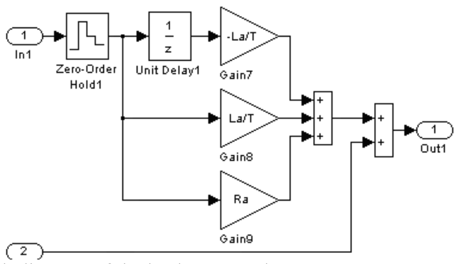

Fig. 2. Block diagram of the back EMF voltage sensor.

The discrete state-space model of the controlled DC drive can be represented as follows:

$$\begin{bmatrix} x_1(k+1) \\ x_1(k+1) \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} v(k) + \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} i_l(k) \quad (4)$$

In order to use the quadratic quality criterion in the process of synthesis, the system error of $e(k) = E_r(k) - E(k)$ should be formulated, where $E_r(k)$ is the respective reference input.

It is assumed that both the reference and disturbance inputs are constant, i.e. $E_r(k) = \text{const}$ and $i_l(k) = \text{const}$. The following equation concerns both error and the respective state variables, which are not outputs [3]:

$$\begin{bmatrix} x_{1e}(k+1) \\ x_{2e}(k+1) \\ x_{3e}(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & a_{11} & -a_{12} \\ 0 & -a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_{1e}(k) \\ x_{2e}(k) \\ x_{2e}(k) \end{bmatrix} + \begin{bmatrix} 0 \\ -b_1 \\ b_2 \end{bmatrix} v_e(k) \quad (5)$$

or

$$\mathbf{x}_e(k+1) = \mathbf{A}_e \mathbf{x}_e(k) + \mathbf{b}_e v_e(k),$$
$$\mathbf{x}_e(0) = \mathbf{x}_{e0},\ k = 0, 1, 2, \dots;$$
$$\mathbf{y}(k) = \mathbf{C}_e \mathbf{x}_e(k),$$

where:

$$\begin{aligned} x_{1e}(k) &= e(k-1) = E_r(k) - E(k-1); \\ x_{2e}(k) &= e(k) - e(k-1) = -[E(k) - E(k-1)]; \\ x_{3e} &= i_a(k) - i_a(k-1); \\ v_e(k) &= v(k) - v(k-1); \\ \mathbf{C}_e &= [1 \quad 0 \quad 0]. \end{aligned} \quad (6)$$

Equation (5) has been used for the synthesis of both an optimal modal digital observer and the respective state controller.

III. SYNTHESIS OF OPTIMAL MODAL OBSERVER

Synthesis of the digital state observer has been carried out by an algorithm shown in [4]. This procedure utilizes the transpositioned additional object [5]:

$$\boldsymbol{\alpha}(k+1) = \mathbf{A}_e^{\mathrm{T}} \boldsymbol{\alpha}(k) + \mathbf{C}_e^{\mathrm{T}} \beta(k) \quad (7)$$

or

$$\begin{bmatrix} \alpha_1(k+1) \\ \alpha_2(k+1) \\ \alpha_3(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & a_{11} & -a_{21} \\ 0 & -a_{11} & -a_{21} \end{bmatrix} \begin{bmatrix} \alpha_1(k) \\ \alpha_2(k) \\ \alpha_3(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \beta(k) \quad (8)$$

The $\mathbf{A}_e^{\mathrm{T}}$ matrix eigenvalues are determined solving the following equation:

$$\det\left\{ \begin{bmatrix} 1 & 1 & 0 \\ 0 & a_{11} & -a_{21} \\ 0 & -a_{11} & -a_{21} \end{bmatrix} - \begin{bmatrix} \chi & 0 & 0 \\ 0 & \chi & 0 \\ 0 & 0 & \chi \end{bmatrix} \right\} = 0 \quad (9)$$

The eigenvalues are as follows:

$$\chi_1 = 1,\ \chi_2 = 0.9985,\ \chi_3 = 0.9813$$

In this case an undesired root of the open-loop system $\chi_1 = 1$ exists, which must be displaced. A location for the closed-loop system root $\mu_1 = 0.1$ is defined, where χ_1 should be placed.

In order to define the observer $\mathbf{H}$ matrix, it is necessary to find the $\mathbf{q}_1$ eigenvector elements, solving the system of homogeneous algebraic equations:

$$(\mathbf{A_e} - \mathbf{I}\chi_i)\mathbf{q}_i = 0 \text{ for } i = 1 \quad (10)$$

For the elements of both eigenvector $\mathbf{q}_1$ and weight matrix $\mathbf{Q}_1$ the following is obtained:

$$\mathbf{q}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix},\ \mathbf{Q}_1 = \mathbf{q}_1 \mathbf{q}_1^{\mathrm{T}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Next products are computed:

$$\mathbf{b}_e^{\mathrm{T}} \mathbf{q}_1 \mathbf{q}_1^{\mathrm{T}} = [1 \quad 0 \quad 0] \text{ and } \mathbf{b}_e^{\mathrm{T}} \mathbf{q}_1 \mathbf{q}_1^{\mathrm{T}} \mathbf{b}_e = 1.$$

The weight coefficient $r_1 = 0.1235$ is calculated as well as the $\lambda_1 = 1.1111$ coefficient.

The respective optimal modal feedback gain is determined:

$$\gamma_1 = \begin{bmatrix} -0.9 \\ 0 \\ 0 \end{bmatrix}.$$

As the undesired eigenvalue is only one in this case, the feedback gain is derived as follows:

$$\gamma^{*\mathrm{T}} = \gamma_1^{\mathrm{T}} = [-0.9 \quad 0 \quad 0].$$

The observer feedback vector is formulated:

$$\mathbf{H} = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = \gamma^* = \begin{bmatrix} -0.9 \\ 0 \\ 0 \end{bmatrix}.$$

The observer equation is as follows [4]:

$$\hat{\mathbf{x}}_e(k+1) = \mathbf{A}_e\hat{\mathbf{x}}_e(k) + \mathbf{b}_e\mathbf{u}_e(k) + \mathbf{H}\Delta\mathbf{e}(k) = \\ = \mathbf{A}_e\hat{\mathbf{x}}_e(k) + \mathbf{b}_e\mathbf{u}_e(k) + \mathbf{H}[y(k) - \mathbf{C}\hat{\mathbf{x}}(k)]$$

or

$$\begin{bmatrix} \hat{x}_{1e}(k+1) \\ \hat{x}_{2e}(k+1) \\ \hat{x}_{3e}(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & a_{11} & -a_{12} \\ 0 & -a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} \hat{x}_{1e}(k) \\ \hat{x}_{2e}(k) \\ \hat{x}_{3e}(k) \end{bmatrix} + \begin{bmatrix} 0 \\ -b_1 \\ b_2 \end{bmatrix} v_e(k) + \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} \Delta e(k)$$

where

$$\Delta e(k) = \mathbf{y}(k) - \mathbf{C}\hat{\mathbf{x}}(k).$$

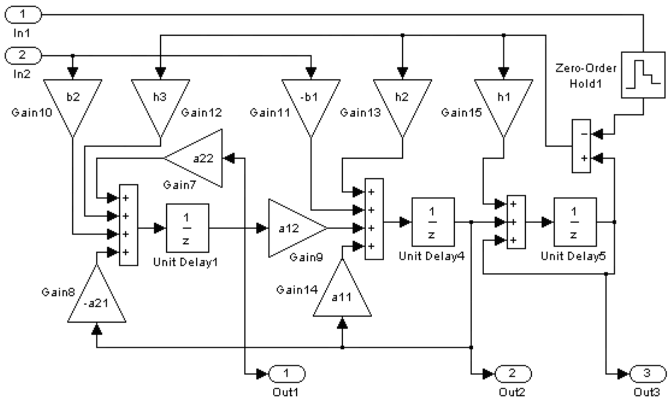

Fig. 3. Block diagram of the synthesized optimal modal observer.

These equations give the state variables valuation. Based on them the respective observer has been developed (Fig. 3).

IV. SYNTHESIS OF OPTIMAL MODAL CONTROLLER

Design of the optimal modal controller has been realized by an algorithm shown in [3]. In this case synthesis is carried out based on equation (5).

The $\mathbf{A}_e$ matrix eigenvalues are as follows:

$$\chi_1 = 0.9813; \; \chi_2 = 0.9985; \; \chi_3 = 1.$$

Among these values an undesired root $\chi_3 = 1$ exists, which should be displaced.

A root location $\mu_3 = 0.95$ of the close-loop system characteristic equation is defined, where the undesired open-loop system eigenvalue χ_3 will be placed.

In order to determine the controller vector $\mathbf{K}$ it is necessary to find the eigenvector $\mathbf{q}_3$ elements, solving the system of homogeneous algebraic equations:

$$\left(\mathbf{A}_e^{\mathrm{T}} - \mathbf{I}\chi_i\right)\mathbf{q}_i = 0 \text{ for } i = 3 \quad (11)$$

The elements of eigenvector $\mathbf{q}_3$ and weight matrix $\mathbf{Q}_3$ are obtained as follows:

$$\mathbf{q}_3 = \begin{bmatrix} 0.0014 \\ 0.9991 \\ -0.0417 \end{bmatrix}, \quad \mathbf{Q}_3 = \begin{bmatrix} 0.0000 & 0.0014 & -0.0001 \\ 0.0014 & 0.9983 & -0.0416 \\ -0.0001 & -00416 & 0.0017 \end{bmatrix}.$$

These products are calculated:

$$\mathbf{b}_e^{\mathrm{T}}\mathbf{Q}_3 = [0.0000 \; -0.0044 \; 0.0002], \; \mathbf{b}_e^{\mathrm{T}}\mathbf{Q}_3\mathbf{b}_e = 1.9255.10^{-5}.$$

For next coefficients the following values are obtained:

$$r_3 = 0.0073 \text{ and } \lambda_3 = 20.$$

As there is only one undesired eigenvalue ($\chi_3 = 1$), the optimal modal feedback gain is derived as follows:

$$\gamma^{*\mathrm{T}} = \gamma_3^{\mathrm{T}} = [0.0158 \quad 11.3846 \quad -0.4747].$$

The feedback vector obtains this form:

$$\mathbf{K} = \gamma^* = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix} = \begin{bmatrix} 0.0158 \\ 11.3846 \\ -0.4747 \end{bmatrix}$$

and control of the following type is formulated:

$$v_e(k) = \mathbf{K}^{\mathrm{T}}\mathbf{x}_e(k) = k_1x_{1e} + k_2x_{2e} + k_3x_{3e} \quad (12)$$

After substitution of $v_e(k)$ in (6), for the optimal modal controller this expression is obtained:

$$v(k) = v(k-1) + k_1x_{1e} + k_2x_{2e} + k_3x_{3e} \quad (13)$$

Based on (13) the respective controller model is constructed, shown in Fig. 4.

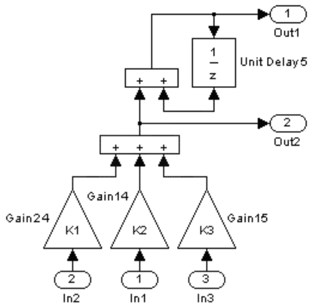

Fig. 4. Block diagram of the optimal modal controller.

Overtaking current limitation has been applied. The respective function is as follows:

$$v_{cl}(k) = v_i + K_mE(k-1), \quad (14)$$

where: v_i is the current limitation initial code; K_m – scale coefficient.

Hence, the control condition in the presence of current limitation will be:

$$v_c(k) = \begin{cases} v(k) & \text{at} \quad v(k) \le v_{cl}(k) \\ v_{cl}(k) & \text{at} \quad v(k) > v_{cl}(k) \end{cases} \quad (15)$$

The control code which should be supplied as input to the chopper is determined by condition (15). In accordance with it the current limitation model is composed (Fig. 5).

Practically, the optimal modal control is achieved through consequent realization of (12), (13), (14) and (15).

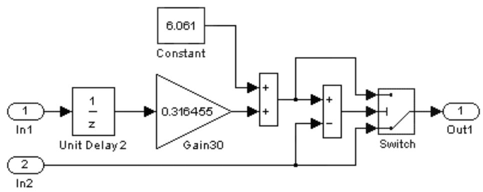

Fig. 5. Model of the applied current limitation.

V. SIMULATION RESULTS

To prove the offered control algorithm functionality some computer simulation models have been developed, using the MATLAB/SIMULINK software package.

The block diagram of the system under consideration is represented in Fig. 6.

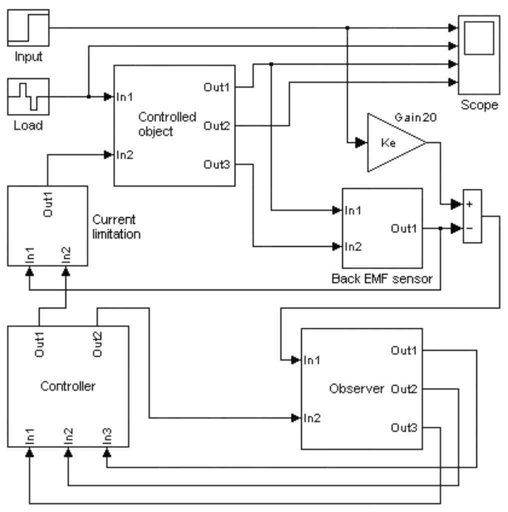

Fig. 6. Block diagram of the complete system under consideration

Fig. 7 shows some simulation results illustrating the behavior of the investigated driving system in the respective transient and steady-state regimes.

The applied quantization period is $T = 0.0001\,\text{s}$ .The reference motor speed is $\omega_r = \omega_{rat} = 115.19$rad/s, and the load torque is equal to the rated value of $T_l = T_{lrat} = 3$Nm. The starting current is limited to the maximum admissible value of $i_{max} = 31.4$ A, which provides for a maximum starting motor torque. The disturbance influence expressed in the load changes has also been illustrated. In the case under consideration these variations are $\Delta T_l = +20\,\%$ and $\Delta T_l = -20\,\%$ consequently.

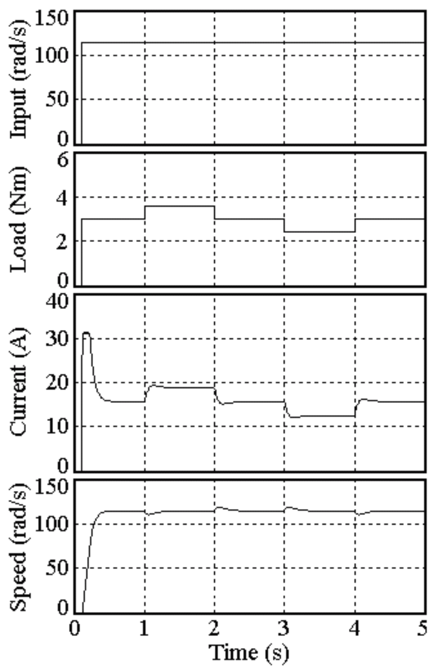

Fig. 7. Time-diagrams illustrating the driving system performance.

VI. CONCLUSION

An approach to sensorless speed control of permanent magnet DC motor drives is presented in this paper. The motor speed has been measured indirectly by the respective back EMF voltage.

The main feature of a system with optimal modal control have been described and discussed.

The analysis carried out shows that the investigated driving system provides for static and dynamic characteristics, analogical to those of the existing sensor version. But its price is lower, due to elimination of the speed sensor.

The results obtained show that the used control method can ensure good performance, which makes it suitable for a variety of applications.

REFERENCES

[1] B. K. Bose, Power Electronics and Motor Drives: Advances and Trends. Academic Press, London, 2006.
[2] M. R. Mikhov, Control System of Electric Drives. Technical University of Sofia, Sofia, 2007.
[3] Ts. T. Gergiev, "Synthesis of Optimal Modal Discrete Controllers," Proceedings of the ICAI, Vol. 2, pp. 119-122, Sofia, 2003.
[4] Ts. T. Georgiev, D. G. Genov, "Synthesis of Optimal Modal Observers," Computer Science and Technologies, Vol. 4, No. 1/2, pp. 51-57, Technical University of Varna, 2006.
[5] R. Iserman, Digital Control Systems. Springer-Verlag, New York, 1981.