

# Mathematical Formulation of Public Electric Transport Scheduling Task for Artificial Immune Systems

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**Abstract** - This paper describes mathematical formulation and application of artificial immune system for scheduling tasks for public electric transport. Artificial immune system is inspired by human immune system to simulate the process of interaction between antigens and antibodies. The task of scheduling in transport system is represented as one of the most well-known flow shop problem. Artificial immune system as a genetic based method is used to solve such task. Mathematical model and algorithm is proposed to create optimal schedule for public electric transport for minimization of electric energy consumption and time. Numerical example shows several steps of algorithm for artificial immune system for scheduling task solution.

## I. INTRODUCTION

Nowadays in metropolis number of vehicles is increasing day by day. It is a reason for a lot of problems for public transport, causing traffic jams, schedule violation, etc. Unexpected standing at crossroads is a reason for often braking with following accelerations myriad times. This is the cause for electric power overconsumption and public transport delay. To provide quick, cheap and comfortable passengers delivery with public electric transport in metropolis there was and will be topical problem. On the one hand, the amount of vehicles in the streets and electricity expenses are going up day by day, traffic jams become from bad to worse, public transport stays in traffic jams longer time and use electricity increasingly, but on the other hand the citizens require public transport as fast and as cheap as possible. Optimal traffic organization, coordination of traffic lights operation with transport moving on the routes according to an optimal schedule will help to solve this actual problem for any big city worldwide [6].

This paper describes a new application of artificial immune algorithm, to create coordinated traffic lights working schedule for optimal electric transport flow on routes. In terms of artificial immune systems, multi-criteria target function is defined as antigen. Schedules which are the solutions of the problem are defined as a set of antibodies. Each antibody represents a sequence of processor to perform all operations of each vehicle on public transport route. Randomly generated schedule is evaluated according to the target affinity function. As genetic based methods, artificial immune systems create new population of antibodies using

specific procedures as diversification, clonal proliferation and hypermutation, which imitate features of biological immune system [10]. Processing continues until the solution is found or predefined stopping criteria is achieved.

## II. PROBLEM FORMULATION

To organize an optimal red green lighting schedule on crossroads is very important for standby time decreasing in public transport. Currently traffic lights in the Riga city are working basically separate from each other. They have not shared control system and have not coordinated red green lighting time along public transport routes.

This paper proposes to use immune algorithm and scheduling theory to create optimal schedule for traffic lights working time at crossroads as solution for traffic jams eliminating in the city. If traffic lights work coordinated according to optimal schedule, this could decrease electric transport standby time, eliminate often breaking and acceleration, finally save electricity.

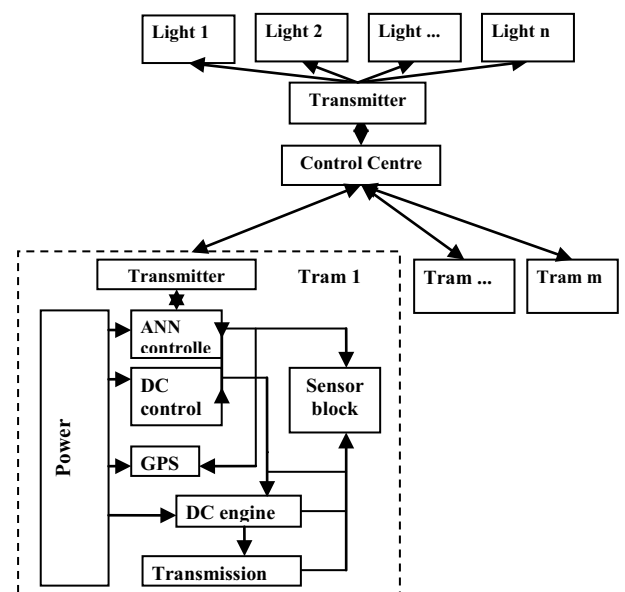


Fig.1. Structure of intelligent control system of electric transport and traffic light

Main tasks of research are to create programmable model for green and red lighting time modeling at crossroads. Main goal of this research is to find out optimal traffic lights working schedule used immune algorithm. Purposed optimal traffic lights and intelligent electric transport organization block diagram is shown in Fig.1.

Traffic lights electrical process and electric transport mechanic and electric process modeling are described below. Traffic lights electrical process control model consists of logical control scheme, switches, transmitter and lamps scheme. Artificial immune system is proposed for traffic light

control device to optimize traffic flow on the set of crossroads [1].

Electric transport control model consists of vehicle control scheme, model of ANN and DC drive model as shown in Fig.2. Electric transport units send signal about current location, schedule etc. to control centre. Artificial immune system of the control centre creates optimal traffic lights working schedule and through transmitter sends signal to traffic lights and in same time relevant information to vehicle's controller which accelerate or brake DC drive according to the received signal [2].

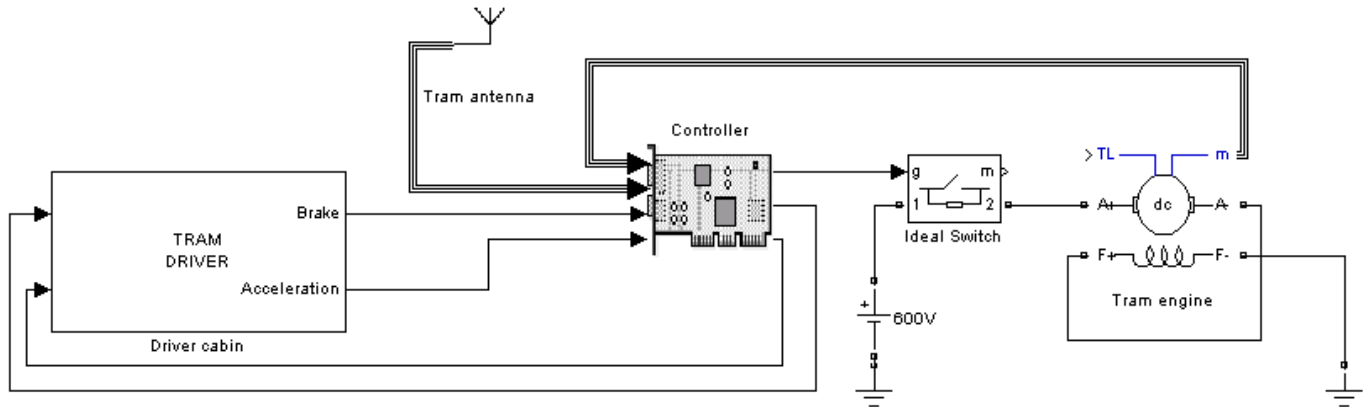


Fig.2. Scheme of intelligent transport control system

### III. MATHEMATICAL MODEL FOR ARTIFICIAL IMMUNE SYSTEM OF TRAFFIC CONTROL

#### A. Nomenclature

The Following definitions are proposed for mathematical model of artificial immune system [10]:

- Processors -  $P = \{p_1, \dots, p_n\}$  - crossroads and streets;
- Jobs -  $TR = \{TR_1, \dots, TR_m\}$  - vehicles;
- Set of sequences -  $S = \{R_1, \dots, R_b\}$  routes, where for each route  $R \in S$ ,  $R = \{o_1, \dots, o_{k_r}\}$ , where
  - o  $k_R \leq n$  - Number of operations for route r;
  - o  $o_i \in P$  - Processor to perform operation i;
- Set of prior operations of each route -  $PR = \{0, o_1, o_2, \dots, o_{k_r-1}\}$ ,  $PR \rightarrow R$
- Each vehicle  $t \in TR$  has a route  $R^t$  assigned to it, where  $R^t \in S$ ;
- Duration for each operation  $o \in R^t$  for each vehicle  $t \in TR$   $D^t = \{d_1^t, \dots, d_{k_r^t}^t\}$ ,  $D^t \rightarrow R^t$ ,  $d \in D^t$ ,  $d \in \mathbb{R}$ ;
- Antibody -  $AB = \{g_1, \dots, g_q\}$  - the schedule for all vehicles, where
- $q = |R^{TR_1}| + |R^{TR_2}| + \dots + |R^{TR_m}|$  - size of antibody is a number of operations in each route of each vehicle;

- $g_i = o_\alpha \in R^t$ ,  $g_j = o_\beta \in R^t$ ,  $i < j \Rightarrow \alpha < \beta$  - sequence of operations, which can not be disarranged for each route  $R^t \in S$  of vehicle  $t \in TR$ ;
- Each gene  $g \in AB$  is represented as a following tuple:  $g = \langle tr, p, p', d \rangle$ , where
  - o  $tr$  - vehicle, which performs operation of gene  $g$
  - o  $p$  - crossroad or street on which vehicle  $tr$ , performs operation of gene  $g$ ;
  - o  $p'$  - prior crossroad or street, where vehicle  $tr$  has performed its previous operation;
  - o  $d$  - duration of operation of gene  $g$ , which performs vehicle  $tr$  on processor  $p$ .
- Each antibody  $AB = \{g_1, \dots, g_q\}$  is represented as the following lists combined with the elements of genes tuples:
  - o  $JL = \{tr_1, \dots, tr_q\}$  - job list, which consists of vehicles operation of each gene in antibody;
  - o  $ML = \{p_1, \dots, p_q\}$  - machine list, which consists of crossroad or streets to perform operation of each gene in antibody;
  - o  $PML = \{p'_1, \dots, p'_q\}$  - predecessor machine list, which contains prior crossroad or street for operation of each gene in antibody;

- $TL = \{d_1, \dots, d_q\}$  - time list, which contains duration of each operation in antibody;
- Antigen – target function
 
$$\left\{ \begin{array}{l} F = f(T, E) \rightarrow \min \\ T = f(AB) = \sum_{i=1}^m T_m \rightarrow \min, \text{ where} \\ E = f(AB) = \sum_{i=1}^m E_m \rightarrow \min \end{array} \right.$$
- $T(AB)$  – total time to fulfil all operations of each vehicle according to schedule AB;
- $E(AB)$  – total electrical energy consumed during fulfilment all operations of each vehicle according to schedule AB;
- $F(T, E)$  – multi-criteria target function, depending on T and E.

#### B. Parameters for immune system

- Antibody population size – z
- Memory pool size - M
- Replacement rate –  $\rho$
- Clonal proliferation rate –  $\kappa$
- Hypermutation rate –  $\psi$
- Donor rate –  $\delta$
- Tournament pressure -  $\gamma$
- Inducing rate –  $\lambda$
- Diversity probability -  $\sigma$
- Bit number in Gene shift -  $\theta$
- Bit number of nucleotide -  $\beta$
- Number of proliferation -  $\eta$

#### C. Additional conditions for traffic control task

Each electric vehicle  $u \in TR$  has following parameters, which depends on time t:

- Current –  $I_u(t)$
- Torque -  $\tau_u(I_u)$
- Acceleration –  $a_u(\tau_u)$
- Deceleration –  $b_u(\tau_u)$
- Velocity –  $v_u(a_u, b_u)$

Therefore, the duration of each operation of vehicle  $u \in TR$  depends on velocity  $v_u$ :

$$D^u = \{d_1^u(v_u), \dots, d_{k_r}^u(v_u)\}.$$

The difference between usual flow-shop scheduling task and traffic control task is that duration of each operation is not predefined and is in functional dependency on the performance of other operations as well as criteria such as conditions of the streets surface, weather conditions, distance

considering ratio among two contiguous vehicles, drivers acquirements, etc.

Due to this reason the evaluation of the schedule may be solved using simulation of each result of artificial immune system for public transport system model, which takes in account the following variable parameters [8]:

- green light duration limits,
- relative number of electric transport in the traffic flow, initial street fullness,
- average length of a vehicle,
- minimal distance between vehicles in traffic jam,
- maximal speed,
- weather – clear, cloudy with rain and heavy rain with reduced visibility, that has influence on
- driver’s reaction time,
- acceleration time to maximal speed.

#### IV. ALGORITHM FOR ARTIFICIAL IMMUNE SYSTEM FOR TASK SOLUTION

##### D. General steps of immune algorithm

Step 1. Random schedule initialization. According to immune algorithm operation sequence, the first step is random initialization of possible antibody (schedule) population.

Generate initial population  $G^0 = \{AB_1, AB_2, \dots, AB_z\}$ , where  $AB_i = \{g_1^i, \dots, g_q^i\}$ ,

$$g_j^i = \text{rand}(R^{TR_1} \cup R^{TR_2} \cup \dots \cup R^{TR_m}), g_j^i \neq g_k^i, \\ i = \overline{1, z}, \quad j, k = \overline{1, q}$$

Step 2. Procedure for simulation of transport system according to schedule. The results of simulation is total time and total energy spent by all vehicles according to schedules:

$$\exists AB \in G^0, \quad T(AB_i), \quad E(AB_i), \quad i = \overline{1, z}$$

Step 3. Evaluation of schedule affinity to target function. Each schedule is evaluated by target function using the results of simulation:

$$\exists AB \in G^0, \quad F_i(T(AB_i), E(AB_i)), \quad i = \overline{1, z}$$

Step 4. Clonal proliferation of the most matched schedule. In the IA scheme, the most matched (Maximum affinity value) schedule derived from the earlier step is chosen for hypermutation after clonal proliferation process.

Step 4.1. Selecting schedule with best affinity for clonal proliferation:

$$F^*(AB^*) = \max(F_1(AB_1), \dots, F_z(AB_z)) \Rightarrow AB_0^*$$

Step 4.2. Proliferating selected schedule  $AB_0^*$  according to proliferation number:

$$CP = \{AB_1^*, AB_2^*, \dots, AB_\kappa^*\}, \quad AB_i^* = AB_0^*, \quad i = \overline{1, \kappa}$$

Step 4.3. Light chain hypermutation in each schedule.  
 Step 4.4. Simulation of hypermutated schedules.  
 Step 4.5. Affinity evaluation for each hypermutated schedules.  
 Step 4.6. Preliminary donor schedule set creation.  
 Step 4.7. Memory pool update.  
 Step 5. Tournament selection for donor schedule. Several schedules according to the predefined tournament size are chosen randomly for competition with the surviving winner being turning into a donor schedule.  
 Step 5.1. Select schedules from preliminary donor schedule set.

Step 5.2. Tournament stage.  
 Step 5.3. If the defined number of schedules is selected, than finish, else go to step 5.2.

Step 6. Germ-line DNA library construction. In IA components from the memory schedules and the donor schedules construct the germ-line DNA library.

Step 7. Gene fragment rearrangement. In IA new schedules are created via gene fragments rearrangement process.

Step 8. Schedule diversification. Matching a large variety of antigens/tasks requires an equal level of diversity in schedule type. In the IA this was achieved by mimicking the following six diversification mechanisms:

- point mutation;
- recombination;
- conversion;
- inversion;
- shift;
- nucleotide addition.

Step 9. Stop criterion. The whole process will stop when the generation equals to a pre-defined number. Otherwise the process reverts to Step 2 for iteration. Finally the best and most diverse solutions are stored in the memory pool.

### V. NUMERICAL EXAMPLE OF IMMUNE ALGORITHM

Numerical example is proposed to show several steps of immune algorithm for scheduling of traffic light. Therefore this example is simplified and duration of each operation is constant.

Let us assume that three crossing streets are given as shown in Fig.4. Streets are split into parts with two types such as crossroads C2 and C4 and parts of the streets between crossroads S1, S3, S5, S6, S7, S8, S9. Three public electric transport routes are given and three electric transport vehicles TR1, TR2 and TR3 moving by these routes.

The task is to create optimal traffic lights working schedule for crossroads C2 and C4 to minimize total time spent by all vehicles to complete their routes.

According to the scheduling theory terms:

- Processors – S1, S3, S5, S6, S7, S8, S9 and C2,C4 - streets and crossroads
- Jobs – TR1, TR2, TR3

Each job has 5 operations – routes:

- $O(\text{TR1}) = \{o11, \dots, o15\}$  and  $ML(\text{TR1}) = \{S5, C4, S3, C2, S1\}$  where operation o11 is processed by S5, etc;
- $O(\text{TR2}) = \{o21, \dots, o25\}$  and  $ML(\text{TR2}) = \{S7, C2, S3, C4, S8\}$  where operation o21 is processed by S7, etc;
- $O(\text{TR3}) = \{o31, \dots, o35\}$  and  $ML(\text{TR3}) = \{S9, C4, S3, C2, S6\}$  where operation o31 is processed by S9, etc.

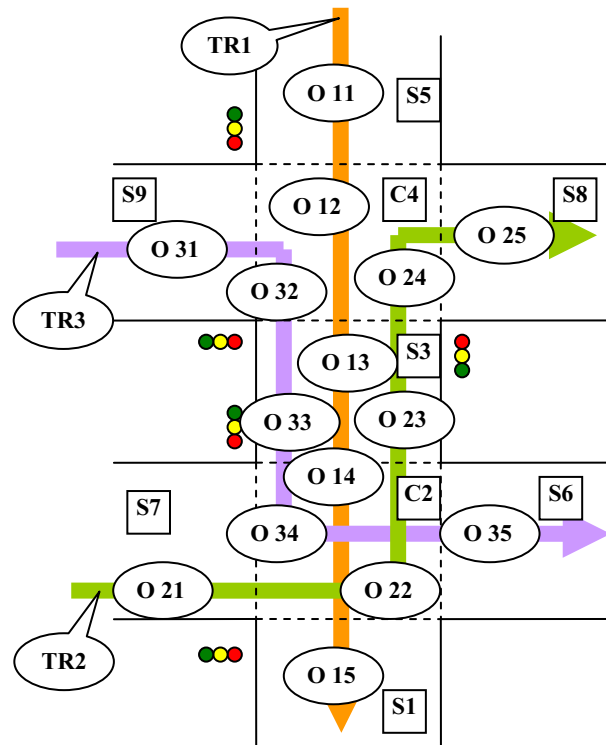


Fig.4. Vehicles moving directions and streets scheme

Operations sequence for transport units are shown in Table I.

TABLE I  
 OPERATIONS SEQUENCE FOR TRANSPORT UNITS

Order	O1	O2	O3	O4	O5
TR1	5	4	3	2	1
TR2	7	2	3	4	8
TR3	9	4	3	2	6

This operations passing duration are shown in Table.

TABLE II  
 OPERATIONS PASSING DURATION

Duration	O1	O2	O3	O4	O5
TR1	6	6	5	2	3
TR2	4	1	7	2	6
TR3	5	2	3	4	7

As Fig.4 demonstrates, C2 and C4 are the crossroads and therefore they have restrictions. Through P2 vehicles coming from S3 and S1 can move simultaneously, but vehicles coming from S3 and S7 moving at same time are restricted. Similar restrictions are for crossroad C4. Through C4 vehicles coming from S3 and S5 can move simultaneously, but vehicles coming from S3 and S9 can not move. The set of „friendly” and „conflict” operators are shown in Table III.

TABLE III  
 SET OF „FRIENDLY” AND „CONFLICT” OPERATORS

S2	3	1	<	7	6
S4	3	5	<	9	8

Step 1. According to the immune algorithm operation sequence, the first step is a random initialization of possible antibody (schedule) population. Schedule contains n\*m genes for n transport units and m operations. In this task the schedule contains 15 genes and randomly generated four schedules AB1, AB2, AB3 and AB4 as shown in Table IV. Each transport unit appears in the schedule m times.

TABLE IV  
 RANDOMLY GENERATED SCHEDULES

AB1	1	2	3	3	2	2	1	1	3	2	1	3	1	2	3
AB2	1	1	1	1	1	3	2	2	2	2	2	3	3	3	3
AB3	2	2	3	3	1	1	2	1	3	3	2	1	1	2	3
AB4	1	2	1	3	3	2	3	1	3	2	3	2	1	2	1

Step 2. Next step is to create related machine list (ML) for each schedule. The related machine lists and corresponding passing time lists (TL) are shown in Table V and Table VI.

TABLE V  
 RELATED MACHINE LIST

ML1	5	7	9	4	2	3	4	3	3	4	2	2	1	8	6
ML2	5	4	3	2	1	9	7	2	3	4	8	4	3	2	6
ML3	7	2	9	4	5	4	3	3	3	2	4	2	1	8	6
ML4	5	7	4	9	4	2	3	3	2	3	6	4	2	8	1

TABLE VI  
 RELATED TIME LIST

TL1	6	4	5	2	1	7	6	5	3	2	2	4	3	6	7
TL2	6	6	5	2	3	5	4	1	7	2	6	2	3	4	7
TL3	4	1	5	2	6	6	7	5	3	4	2	2	3	6	7
TL4	6	4	6	5	2	1	3	5	4	7	7	2	2	6	3

TABLE VII  
 PREDECESSOR MACHINE LIST

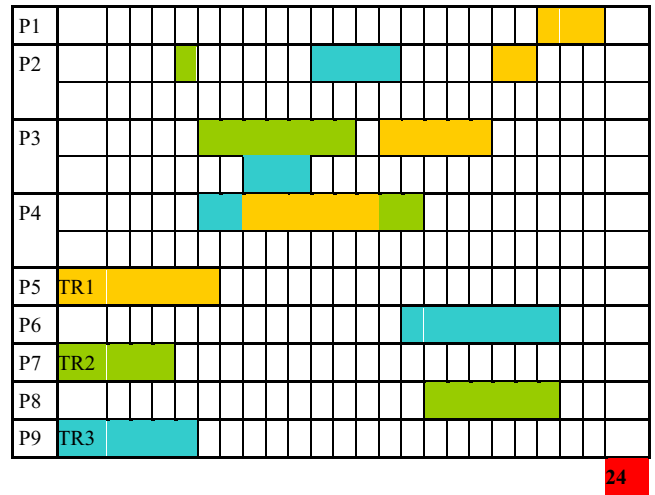
PML1	0	0	0	9	7	2	5	4	4	3	3	3	2	4	2
PML2	0	5	4	3	2	0	0	7	2	3	4	9	4	3	2
PML3	0	7	0	9	0	5	2	4	4	3	3	3	2	4	2
PML4	0	0	5	0	9	7	4	4	3	2	2	3	3	4	2

According to the restrictions predecessor machine list is created, set of previous operators, which from TR1, TR2 and TR3 are coming. Predecessor machine list dates are used calculating value of makespan. Predecessor machine list is shown in Table VII.

$$AB_i = \frac{obj_i}{SC_i} \text{ and } obj_i = \frac{\min\{makespan_i | i = 1, 2, \dots, N_{Ab}\}}{makespan_i}$$

where makespan<sub>i</sub> indicates makespan value of the ith schedule and obj<sub>i</sub> is its associated normalized value. To find value min makespan are necessary create Gantt chart for each schedule. As shown in Table VIII min makespan to operate schedule complete is 24 seconds.

TABLE VIII  
 MIN MAKESPAN CALCULATING



The relationship among the schedules is evaluated according to the similarity count SC<sub>i</sub> expressed as:

$$SC_i = \frac{\sum_{j=1}^{N_{Ab}} count_{ij}}{N_{Ab}}, \text{ i, j} = 1, 2, \dots, N_{Ab};$$

with

$$count_{ij} = \frac{\sum_k^{nm} Ab_{ij}^k}{n * m}$$

where the similarity count at the k locus among schedules Ab<sub>i</sub> and Ab<sub>j</sub> is expressed as:

$$AB_{ij}^k = \begin{cases} 1 & \text{if the jobs at the k locus of } Ab_i \text{ and } Ab_j \text{ are identical} \\ 0 & \text{else} \end{cases}$$

Ab<sub>ijk</sub> calculating results, values of count<sub>ij</sub> are shown in Table IX. Values of obj<sub>i</sub> and Ab<sub>i</sub> are shown in Table X.

TABLE IX  
ABIJK CALCULATING AND VALUES OF COUNTIJ

Ab similar														sum	count
AB1#AB2	1	0	0	0	0	0	0	0	1	0	1	0	1	4	0,266667
AB1#AB3	0	1	1	1	0	0	0	1	1	0	0	0	1	7	0,466667
AB1#AB4	1	1	0	1	0	1	0	1	1	1	0	0	1	9	0,6
AB2#AB3	0	0	0	0	1	0	1	0	0	0	1	0	0	4	0,266667
AB2#AB4	1	0	1	0	0	0	0	0	0	0	0	0	0	2	0,133333
AB3#AB4	0	1	0	1	0	0	0	1	0	0	0	0	1	5	0,333333

TABLE X  
VALUES OF OBJI AND ABI

	SC	Obj	AgAb
SC1	0,333333	0,96	2,88
<b>SC2</b>	<b>0,166667</b>	<b>0,77</b>	<b>4,62</b>
SC3	0,266667	1	3,75
SC4	0,266667	0,83	3,1125

A higher affinity means that the schedule has a higher activation with an antigen and a lower similarity with the other schedules. Consequently, highest affinity has second populated schedule AB2.

## VI. CONCLUSIONS

Application of artificial immune algorithm in scheduling tasks show great promise. To solve multi-criteria scheduling problem for public electric transport flow optimization with artificial immune algorithm application is plan for the nearest future and expecting goal of this investigation is decreasing make span time.

Create mathematical formulation and investigate immune algorithm application for scheduling tasks conclusions for the present are:

- On biological immune system based artificial immune algorithm can be applied to create coordinated optimal traffic lights working schedule
- Transport flow optimization tasks solving result mostly will approximate, due to several transport flow dependent criterions could not be defined unequivocal
- Artificial immune algorithm can be applied to solve multi criteria transport flow optimization tasks
- Coordinated optimal traffic lights working schedule can be applied for minimize electricity consumption and tasks for transport units standby time decreasing.

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