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Approach to Stochastic Modeling of Power Systems

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***Abstract.* This paper presents an approach to modeling power system that contains sources of stochastic disturbance. It is based on frequency analysis of linearized model of power system. Power system dynamic properties are accounted by equivalent transfer functions of machines and their control equipment. This will allow more accurate calculations for different analysis tasks. Methodology of system linearization is proposed and results of linearized model test are delivered.**

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***Keywords*: Frequency domain analysis, stochastic modeling, uncertainty.**

I. Introduction

Deterministic models are still widely used in practice of mode control of complex power systems, short and long-term planning, transient analysis, development of control and protection systems.

However real power system always remains in infinite transient process, caused by permanent change of load and generation value. Faults, line and generating unit tripping also contribute to this process. Deterministic calculation of power system state gives us single point in the space of possible states. To cover full range of states one should vary different parameters of the system. This leads to excessive time and resources consumption.

Thus, stochastic nature and often incompleteness of input data is obvious. We can mark the following stochastic factors:

- load value;
- generation value (i.e. generation instability as a result of speed regulator action and frequency regulation, power variation of renewable power sources, unit tripping);
- changes in power grid;
- changes of system parameters caused by environment influence.

There are several general approaches on how to account stochastic in power system analysis. First is to represent some system variable by probabilistic values, i.e. define them with mean value, standard deviation and so on. To obtain probabilistic characteristic of load flow number of methods were developed including Monte Carlo method, approximation methods etc [1]-[4].

Another approach is based on building scenarios [5]-[10]. In these methods time is divided into periods and it is assumed that parameters considered to be stochastic take discrete values from some range at the end of each period. Sequence of parameter value at each time period makes a scenario that may be assigned probability of its realization. All possible scenarios give a scenario tree.

Advantage of this approach is that it allows to account control actions applied to power system as a reaction to deviation, i.e. transfer load from on power plant to another for power flow optimization.

However, system dynamics (machine inertia, speed of regulation) are not accounted as at each time point only steady state calculation is made.

The approach proposed intended to allow stochastic modeling of power systems with account of its dynamic behavior.

II. Frequency Analisys Based Approach

In proposed problem formulation power system is considered as dynamic object, which is subject to stochastic disturbances. In this case system is represented by system of differential equations.

Thus dynamic response of generators and other controllable devices is accounted. This will allow to reach more accuracy in power flow optimization task, assess probability of dangerous or undesirable state parameter deviations and get more reliable results for state estimation task.

Methods developed in control theory allow to calculate statistical characteristics of linear dynamic system response to fluctuating input signal.

Namely if dynamic system is represented by transfer function $K(\omega)$, power spectral density of input (S_x) and output (S_y) signals are connected by equation

$$S_y(\omega) = \left|K(\omega)\right|^2 S_x(\omega). \qquad (1)$$

The idea of using linear dynamic systems analysis for stochastic modeling of power systems was earlier proposed in the number of papers [11], [12].

Power system as dynamic object is non-linear. To be able to use these methods in power system analysis it is necessary to develop methodology of reduction of power system dynamic model to linear form. Thus we must solve the problem of the following types of nonlinearities:

1. Nonlinear algebraic equations
2. Nonlinearities of control systems (e.g. saturation, dead zones)
3. Nonlinearities related to operational constraints.

Nonlinearities in algebraic equations manifest itself mainly as nonlinear relation between currents, voltages and powers, which are input and output parameters of parts of the model.

Nonlinearities in control systems are caused by limitations which are placed on level of (intermediate) signals in those systems, and presence of “dead zones”.

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Under operational constraints are considered those, caused by technical characteristics of equipment and power system: limitation on active and reactive power generated, transition line capacity, etc.

At first stage of research described in this paper the following simplifications are made: the model describes only active power balance and active power control; only nonlinearities of first type mentioned above are accounted.

III. MODEL DEVELOPMENT

In this section we'll describe linearized power system model.

The dynamics of generators is represented by the following equation:

$$\tau_J p\omega = P_M - P_E, \tag{2}$$

where P_M – mechanical power, P_E – electrical power, τ_J – inertia time constant, ω – rotation speed, p – Laplace operator.

Mechanical power supplied we represent as

$$P_M = W_\omega(p)\omega, \tag{3}$$

where W_ω – equivalent transfer function of the governor.

Using (2) and (3) we can produce equation for generator

$$\omega = -\frac{P_E}{p\tau_J - W_\omega(p)} = W_G(p)P_E, \tag{4}$$

where W_G – equivalent transfer function for generator.

For power grid model we start with nonlinear power balance equation (for i-th node)

$$P_{Gi} - P_{Li} = \\ = U_i^2 y_{ii} \sin\alpha_{ii} + \sum_{\substack{j=1 \\ j\neq i}}^{n} U_i U_j y_{ij} \sin(\delta_{ij} - \alpha_{ij}), \tag{4}$$

where P_{Gi}, P_{Li} – node generation and load power, U_i, U_j – node voltages, δ_{ij} – phase angle between two nodes, y_{ii}, y_{ij} – self and mutual admittance absolute value , α_{ii}, α_{ij} – self and mutual admittance complimentary angle, n – number of nodes.

First order linearization at the point of initial system state gives the following matrix equation for power grid

$$\|\Delta P\| = \|B\| \cdot \|\Delta\delta\|, \tag{5}$$

where B is a matrix of coefficients.

After transformations we can write equation for generator node electrical power

$$\|\Delta P_E\| = \|W_{GG}\| \cdot \|\Delta\delta_G\| + \|W_{GL}\| \cdot \|\Delta P_L\|, \tag{6}$$

Where $\Delta\delta_G$ – array of generator e.m.f. angle deviations, ΔP_L – array of load power deviation, W_{GG} and W_{GL} – matrices of coefficients.

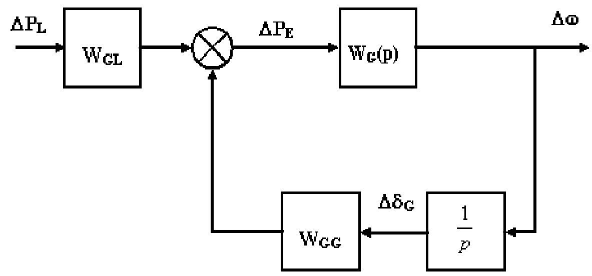

Fig. 1. The model of power system after linearization.

Using (4) and (6) gives us the model of entire power system, which diagram is shown at Fig. 1. It gives connection between load power deviation and generator rotation speed deviation. This connection may be represented by equivalent transfer function. It in turn allows to use (1) to calculate reaction of the power system.

IV. TEST SYSTEM RESEARCH

The model developed was tested on two machine power system. Its equivalent scheme is shown at Fig.2.

It contains two generators each rated to 187 MVA. Generator 1 load at steady state is 90 MW. Generator 2 acts as swing bus. Generator 1 is equipped with simple proportional speed control unit with gain k_p and time constant T, thus transfer function of turbine governor is

$$W_\omega = \frac{k_P}{1+pT}. \tag{7}$$

Load is modeled as a sum of two parts: constant and variable. Constant part is represented by resistance.

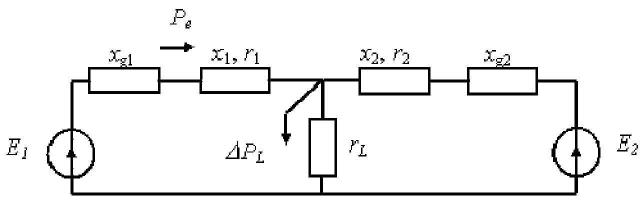

Fig. 2. Test system scheme.

The model was tested by comparison of results produced by linearized model and results of time domain simulation of the same power system.

At first frequency characteristics were compared. In this experiment variable part of load power was changed sinusoidal. The ratio of generator rotation speed deviation

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amplitude to load power deviation amplitude gives us absolute value of transfer function on specific frequency.

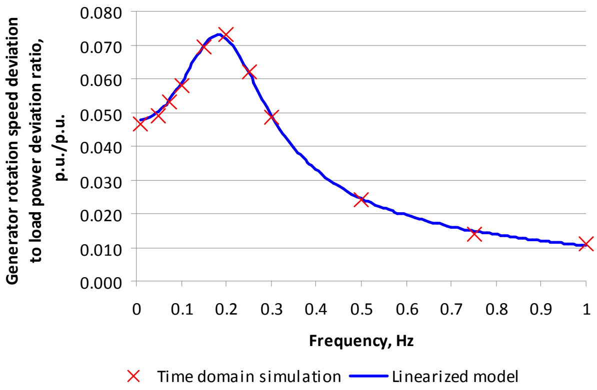

Fig. 3. Frequency characteristics of the test system.

At Fig. 3 we show frequency characteristics obtained from time domain simulation (amplitude of load deviation is 30 MW) and from linearized model analysis. The figure shows good matching of these curves.

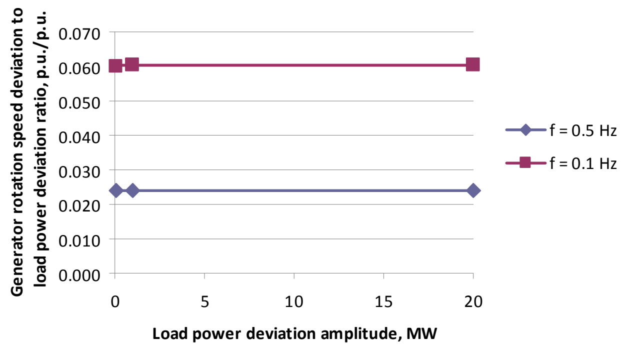

Fig. 4. Dependency of generator rotation speed deviation amplitude to load power deviation amplitude ratio on value of deviation.

To identify linearization errors we made series of calculations with different load power deviation amplitude. The results are shown at Fig. 4.

The figure shows that linearization of power balance equations doesn't cause any significant error, at least if system remains stable after disturbance.

The model developed was tested with sample stochastic process representing load power fluctuation. It was synthesized by random changing of load power every 5 s.

Full duration of the process is 300 s.

Series of calculations was performed with different time constant T in turbine governor transfer function (7).

Power spectral density (PSD) for load power deviation was estimated with Welch method. Then using (1) we calculate PSD for rotation speed deviation as system response.

Time domain simulation with load changing according to synthesized profile was used as reference.

PSD for two cases are shown at Fig. 5.

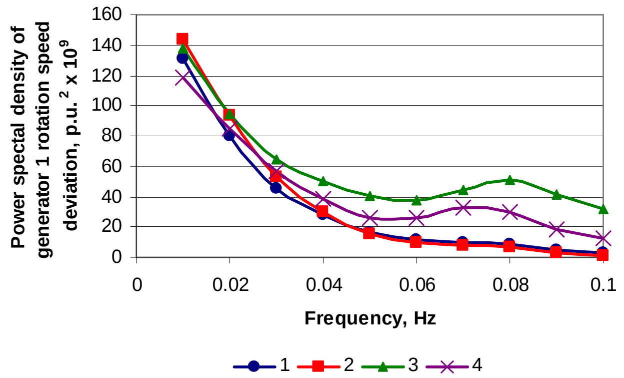

Fig. 5. PSD of generator rotation speed deviation. Curves 1 and 2 – linear model and time domain simulation results respectively for time constant T = 0.8 s; curves 3 and 4 – the same curves for T = 3.0 s.

This diagram shows that results that linear model produce don't match exactly to those of time domain simulation and the error rises with time constant *T*, i.e. error depends on speed of the process – slower processes produce greater error.

Previously we showed that linearization itself doesn't give any noticeable error. Thus difference in PSD curves is likely caused by inaccuracy of estimation algorithm.

TABLE I

STANDARD DEVIATION FOR GENERATOR ROTATION SPEAD

Turbine governor time constant *T*, s	Standard deviation, rad/s	
	Linear model	**Time domain simulation**
0.8	0.015	0.016
3	0.019	0.017
5	0.026	0.023
10	0.042	0.035

If we integrate PSD with respect to frequency we'll get standard deviation for value under consideration. Table I shows standard deviation of rotation speed for different time constant value. We see the same tendency of getting bigger error for bigger time constant.

V. CONCLUSIONS

Approach proposed allows to account dynamic properties of power system in stochastic model. It is expected that that approach will give more accurate results than existing methods if speed of change of fluctuating parameters is in the same range as speed of transients in the system.

To get benefits of this approach linearization of the model is required. As a result of research we may conclude that grid equations first order linearization doesn't give any noticeable error. Errors of PSD estimation are more significant and attention shall be paid to estimation algorithm to get accurate results.

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