The Use of Kramers-Kronig Relations for Verification of Quality of Ferrite Magnetic Spectra

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Abstract – The complex initial permeability (CIP) as a function of frequency is one of the main properties of ferrites. This characteristic (CIP) is measured experimentally, therefore there can be found noisy, doubtful or incomplete parts of the spectrum. Thus there is a need for a method of evaluation of quality of CIP. In this article for evaluation of the quality of experimental CIP spectra of polycrystalline ferrite materials the KKR (Kramers-Kronig relations) are used. In order to apply KKR to experimentally measured data (i.e. data with finite limits) the method of transforming these integral relations into summation relations with finite limits is developed and described. This method can be used only for CIP given over the wide frequency rage, so that the imaginary part of CIP is fully presented. Using KKR with the help of CIP spectra model (based on the effects coming from polycrystal grain sizes and defects distribution) partly removes aforementioned limit. Thus with the help of the model we can also make CIP spectra reconstruction (in cases when CIP is noisy or incomplete) and CIP spectra decomposition.

Keywords – mathematical model, mathematical analysis, permeability, magnetic properties, ferrites.

I. INTRODUCTION

The research of magnetic components, used in power electronics, remains topical nowadays due to extensive use of electronics in everyday life. One of the magnetic component characteristics, which represents losses in ferrite materials (thus, can be used for improvement of power supply efficiency), is the complex initial permeability (CIP) spectrum \( \mu(f) = \mu'(f) - j\mu''(f) \) as a function of frequency \( f \), where \( \mu'(f) \) and \( \mu''(f) \) stands for real and imaginary parts of \( \mu(f) \) respectively.

The experimental magnetic spectra, as a complex function, are measured by different scientists for a long time. Thus there are different approaches and methods, but each approach has its limitations. Thereby there can be found unclear, noisy or incomplete CIP spectra. And if we can repeat our own measurements again, then the spectra taken for analysis from different other author’s publications should be analyzed as they are, due to inability of performing measurements of original samples. This can lead to different interpretations of results.

Such complex data can be evaluated by dispersion relations or Kramers-Kronig relations (KKR). The real part of a complex characteristic can be calculated by means of KKR from the imaginary part - and vice versa. In this way - the complex function can be reconstructed even if one part is available over the full frequency range. Thus the KKR do not provide any physical nature analysis, but are used in the cases, when there is a need for mathematical evaluation of a complex function. Such cases can differ: in electric circuit theory – dispersion relations connect gain frequency dependence and phase; the impedance data over limited frequency domain can be evaluated with KKR [14, 15]; refraction of light and absorption in dispersing medium in optics [16], even application of KKR to S-parameters measured by vector network analyzer [17], etc. The special significance of these relations is in quantum electrodynamics and elementary particle physics, where data about one aspect of the phenomenon gives an indication to other. In this paper the Kramers-Kronig relations will be used in order to evaluate the quality of the experimental CIP and precise measured CIP data.

II. THE ORIGINATE OF DISPERSION RELATIONS

The mathematical basis for dispersion relations comes from Cauchy integral (1) for complex value \( z \): if function \( f(z) \) is analytical in closed domain \( G \), which is limited by closed loop \( \gamma \) [18], then:

\[
\oint_\gamma f(z)dz = 0. \tag{1}
\]

In accordance with (1), the analytical function \( f(z') \) can be expressed in terms of the closed loop \( \gamma' \) integral, which is limited by \( G \) domain, where \( f(z) \) is analytical if \( z' \) lies inside the closed loop [18]:

\[
f(z') = \frac{1}{2\pi i} \oint \frac{f(z)}{z - z'}dz. \tag{2}
\]

The integral transforms to zero if \( z' \) lies outside the closed loop.

Assuming, that \( z' = x' + iy' \) and \( y' \to 0^+ \) we can get the dispersion relations basic formula [19]:

\[
f(x') = \frac{1}{\pi i} \int_{-\infty}^{\infty} \frac{f(x)}{x - x'}dx. \tag{3}
\]

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Now, separating \( f(x') \) into the real and imaginary part we can get:

\[
\begin{align*}
\mu(x') &= \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{u(x)}{x-x'} dx, \\
v(x') &= -\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{u(x)}{x-x'} dx,
\end{align*}
\]

where \( u(x') \) and \( v(x') \) are \( f(x) \) real and imaginary parts.

### A. DISPERSION RELATIONS IN ACCORDANCE WITH CIP

The frequency dependence of magnetic spectra of ferrites is one of the most important parameters of magnetic components. The main idea of this article is to analyze possibilities of application of dispersion relations to magnetic spectra of ferrites.

In 1927 H. Kramers [9] and in 1926 R. Kroning [4] formulated relations between real and imaginary parts of complex function:

\[
\begin{align*}
\chi_1'(v') - \chi_1(\infty) &= \frac{2}{\pi} \int_{0}^{\infty} \frac{\chi_1'(v)\chi_1(v)}{v^2 - v'^2} dv, \\
\chi_2'(v') &= -\frac{2\nu}{\pi} \int_{0}^{\infty} \frac{\chi_2(v)\chi_1(v)}{v^2 - v'^2} dv,
\end{align*}
\]

where \( \chi_1'(v') \) and \( \chi_2'(v') \) - are real and imaginary part of frequency dependent function; \( \nu \) - frequency of sinusoidal magnetic field (which is applied to the device under test); \( v' \) - frequency for which the values of \( \chi_1'(v') \) and \( \chi_2'(v') \) are calculated.

The KKR, written for the real and imaginary CIP components can be presented as follows [20]:

\[
\begin{align*}
\mu'(f) - 1 &= \frac{2}{\pi} \int_{0}^{\infty} \frac{\mu'(x)}{x^2 - f^2} dx, \\
\mu''(f) &= -\frac{2}{\pi} \int_{0}^{\infty} \frac{\mu'(x)}{x^2 - f^2} dx.
\end{align*}
\]

Now, from (12) and (13) sum we can get the \( m_i \):

\[
m_i = \frac{y(b_i) - y(a_i)}{b_i - a_i}.
\]

Subtracting \( y(b_i) - y(a_i) \) and inserting (14) we obtain \( c_i \):

\[
c_i = \frac{y(a_i) - y(b_i)}{b_i - a_i}.
\]

The final step is to substitute the integration in (10) and (11) with summation. In order to do this – we can use the integral solving relations [10] and write the final form of the KKR as [20]:

\[
\begin{align*}
\mu'(f) - 1 &= \frac{2}{\pi} \sum_{i} \left( m_i (b_i - a_i) - \frac{m_i f}{2} \ln \left( \frac{f - a_i}{f - b_i} \right) \right) + \frac{c_i f}{2} \ln \left( \frac{b_i^2 - f^2}{a_i^2 - f^2} \right), \\
\mu''(f) &= \frac{2}{\pi} \sum_{i} \left( -\frac{m_i f}{2} \ln \left( \frac{f^2 - b_i^2}{f^2 - a_i^2} \right) + \frac{c_i f}{2} \ln \left( \frac{f - a_i}{f - b_i} \right) \right).
\end{align*}
\]

The equations (16) and (17) now represent KKR in summation form, this allows for finite frequency limits of experimentally measured data.

### III. PRACTICAL USE OF KKR

As it was mentioned above – the KKR can be used for evaluating the experimental CIP quality. For instance – application of KKR to CIP of our measured MnZn ferrite (material EPCOS TDK – 37, dimensions R12.5x7.5x5 (mm) and initial permeability of 6500) presented in Fig. 2. As it can be seen – there are no significant difference in measured results (shown as solid lines) and KKR evaluation of \( \mu'(f) \) and \( \mu''(f) \) (albeit there is difference in given and measured permeability values). Thus it can be assumed that CIP is measured correctly and can be used for further analysis.

The same ferrite, but with larger dimensions – R40x24x16 (mm), begins to show the attributes of dimensional resonance (such as \( \mu''(f) \) maximum frequency is lowered and resonance...
is more pronounced) [13]. In this case the application of KKR to measured permeability curve (Fig. 3) also shows good correlation, thus the measured data is reliable.

IV. THE USE OF KKR FOR CIP SPECTRA RECONSTRUCTION

However, sometimes a necessity for analyzing the spectra of other author’s publications appears. Therefore the measurements cannot be performed again in order to precise noisy or doubtful areas of the CIP. For example, the CIP spectrum of Fe_{80}Si_{10}B_{10} measured with different sample thicknesses by Dok Won Lee, et al., in [5]. Fig. 4, seems doubtful since the value of \( \mu''(f) \) in all of the three presented in Fig. 4 measurement cases stays practically the same, while the values of \( \mu(0) \) change considerably. More to it the measured data within lower frequencies region (from 1 to \(~50\) MHz) is noisy, especially for the real part of permeability, so it is hard to find the correct value. The KKR evaluation (Fig. 5, green line) of the \( \mu'(f) \) and \( \mu''(f) \) show, that in [5] measured CIP data are inaccurate.

A. CIP SPECTRA RECONSTRUCTION WITH THE HELP OF SPECTRA MODELING

The example of noisy CIP data of Ni_{0.2}Zn_{0.8}Fe_{2}O_{4} ferrite film, found in [6], additionally is not measured fully. Here the chosen measurement frequency represents CIP at \( 2\times10^{3}<f<3\times10^{6} \) Hz. Fig. 6. Thus, for correct evaluation this spectrum needs to be extrapolated till zero. For such a case a model based on grain distribution effects was developed.

The model is discussed in details in [1, 2]. Briefly – the model uses concept that log normally distributed ferrite grains can be represented by low loss oscillators. Thus, the whole \( \mu''(f) \) can be made from the sum of absorption curves of these low loss oscillators. Performing the statistical averaging of permeability of grains and making several assumptions we finally can get the expression for the \( \mu''(f) \) as:

\[
\mu''(f) = \mu_{\text{max}} \exp[-(\log f - \log f_{\text{in}})^2/2\sigma^2],
\]

where \( \mu_{\text{max}} \) and \( f_{\text{in}} \) are the maximum amplitude of \( \mu''(f) \) and its corresponding frequency; \( \sigma = 2\sigma_{D} \) for ferrites without in-grain defects and \( \sigma = \sigma_{D} \) for technical quality ferrites; \( \sigma_{D} \approx 0.22 \) [3]. With the help of Eq. (18) symmetrical \( \mu''(f) \) (Fig. 7, \( \sigma = 0.44 \) ) can be modelled.

Fig. 3. KKR application to MnZn TDK-37 ferrite sample with dimensional resonance [13]

Fig. 7. Possibilities of the use of Eq. (18) for modelling symmetrical CIP (with \( \sigma = 0.44 \)), and asymmetrical CIP (with \( \sigma_{a} = 0.2 \) and \( \sigma_{b} = 0.6 \))[20]
The value of $\sigma$ for experimental spectrum can be found from condition that it and its theoretical approximation should be equal at some point at frequency $f_a \neq f_w$, thus getting:

$$\sigma = |\log(f_a/f_w)| / \sqrt{2\ln[\mu''_{\text{max}}/\mu''(f_a)]}. \quad (19)$$

Now, considering, that most of experimentally analyzed ferrites do not have the ideal microstructure, and due to intragrain defects the CIP of these ferrites show asymmetrical attributes the model can be modified. For this purpose Eq. (18) can be used with two values of $\delta$: one, $\sigma_a$, for frequencies $f_a < f_w$ and other, $\sigma_b$, for $f_a > f_w$. With these two values of $\sigma$ the asymmetrical CIP can be modelled.

Returning to Fig. 6 – where the model was applied, controlling the slope of the imaginary part at frequencies $f > 8 \times 10^6 \text{Hz}$. The slope was chosen so, that slope of the real part calculated with KKR (at frequencies $3 \times 10^6 < f < 2 \times 10^8 \text{Hz}$) will be the same as the experimental one. The analysis of this spectrum showed, that real part was measured successfully and imaginary part is reliable up to $1 \ldots 2 \text{GHz}$ (w/o taking in account the noise).

The above example of incomplete spectrum does not stand as solitary instance. The other incomplete spectrum is presented in [11]. The CIP data of Ni$_{0.55}$Zn$_{0.45}$Fe$_2$O$_4$ ferrite measured by Islam R. (Fig. 8) is cut at about $f = 10^2 \text{Hz}$. More to it – the $\mu''(f)$ part shows high increase of amplitude at lower frequencies, but, at the same time, $\mu'(f)$ part does not show any additional changes of amplitude. This doubtful area should be checked with KKR, but for the correct evaluation of the curves we need the full spectrum. So, we prolonged the $\mu'(f)$ part till $5 \text{MHz}$ and applied KKR. As it can be seen there is no any increase of amplitude at lower frequencies for the $\mu_{\text{KKR}}(f)$, so it can be concluded that this increase of amplitude can be a measurement error.

The example of incomplete, noisy CIP, but which provides reliable data - can be chosen from 3M Company flux-field directional materials (AB5016RF ferrite [12]). The AB5016RF CIP (Fig. 9) is cut at the frequency $f = 800 \text{MHz}$, and there is slight noise at lower frequencies up to $10 \text{MHz}$. Once again with the help of the model and several KKR iterations the curves were prolonged up to $10^8 \text{Hz}$, and precised. The analysis shows that CIP data is precise and reliable; the data within the noisy area of the CIP was also precised with the help of KKR and can be seen in Fig. 9 (the parameters of the model for the in this case are: $\sigma_a = 0.35$, $\sigma_b = 0.79$).

V. DECOMPOSITION OF CIP

The nature of KKR – its ability to calculate the missing part of complex function if the other is present – can be used for CIP spectra decomposition.

Basically - there can be found three dispersion regions within full CIP spectrum (Fig. 10), more frequently are encountered the domain wall (DW) and natural spin resonance (NSR) regions [8]. The contribution of DW and NSR in total CIP spectrum is hard to measure directly. Thus, in order to test the hypothesis that KKR can be used for spectra decomposition the spectrum measured by J. Slama, et. all was chosen. [7] (Fig. 11). This spectrum has two resonances: DW resonance near $f = 90 \text{MHz}$ and NSR near $f = 1.6 \text{GHz}$.

The separation of DW and NSR parts of this spectrum can be done as follows: firstly we need to use the model provided by [1, 2], and reconstruct the DW resonance curve without spin resonance (Fig. 12); then with the help of KKR the real part of DW resonance can be obtained; finally we can evaluate the DW part contribution to whole CIP.

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The parameters of the model for the DW component case are:

\[ \sigma_a = 0.35, \sigma_b = 0.79. \]

![Fig. 12. KKR evaluation of DW component of the measured [7] CIP spectrum.](image)

Identically we can get the real part of the spin resonance (Fig. 13).

This example proves the hypothesis that KKR can be used for spectra decomposition at least in the cases when DW and NSR resonances can be easily identified.

![Fig. 11. CIP of NiZn ferrite [7] with DW and NSR resonances.](image)

VI. RESULTS

This investigation proved that application of Kramers-Kronig relations to the CIP data form great mathematical tool for evaluating the experimental CIP spectra quality, helps to identify the real values of noisy spectrum and points to doubtful data (and, such wise, precise losses in ferrites).

The proposed approach allows using the KKR for the finite frequency region of CIP. The only limitation for this approach is to ensure that analyzed spectrum is presented fully within the given frequency region.

The above mentioned limitation we can overcome by combined use of the KKR and the model [1, 2], which further enhances KKR possibilities. With the help of the model we can reconstruct incomplete spectra (in the case if the measurement equipment limitations does not allow for full spectra measurements, or in case of analyzing the spectra of other author’s publications), as well as for spectra decomposition.

The results of spectra decomposition showed that very complicated measurements can be successfully substituted by application of the KKR together with the model to the CIP and evaluation of the each part of CIP separately.

All these practical applications of KKR prove that use of Kramers-Kronig relations on experimental CIP data is reliable and indispensable in the cases when the studied data is doubtful or is not fully presented.

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